FINDING THE DISCOUNT RATE FOR A PRIVATE FIRM USING PUBLIC COMPARABLES

Lynda S. Livingston, University of Puget Sound

ABSTRACT

Determining the cost of equity is one of the most difficult problems in corporate finance. In this paper, we present a simple estimation example using an internet start-up company. We use public firm comparables for beta, making adjustments for leverage using Harris and Pringle's (1985) assumptions, rather than Hamada's (1972). While we consider adjustments for size, unsystematic risk, and illiquidity, we argue that significant adjustments to a Capital Asset Pricing Model approach using public comparables may be unnecessary.

JEL: G10, G11

KEYWORDS: CAPM, Equity Valuation, Private Firm Valuation

INTRODUCTION

In their influential corporate finance textbook, Brealey, Myers, and Allen (2011) say that "[e]stimating the opportunity cost of capital is one of the hardest tasks in financial management" (page 8). In apparent agreement, Pinto *et al.* (2010), writing for the practitioner and academic audience of the CFA Institute, added a full chapter on return concepts to the second edition of their book *Equity Asset Valuation*. This book also contains a full chapter on the valuation of private companies, whose potential illiquidity and opacity compound the problems analysts face with public companies. In this paper, we present a straightforward approach to estimating the cost of equity for a private firm, using an internet start-up company as an example. Students in introductory corporate finance courses may find the cost of equity easier to understand if they work through a real example using real data, while more advanced students can benefit from a consideration of size and illiquidity premiums.

The cost of capital has spawned robust debate ever since Modigliani and Miller (1958) presented their "capital structure irrelevance" propositions. For example, theorists have argued about the importance of the debt ratio to capital costs: how is the cost of capital affected if debt is rebalanced annually, continuously, or held constant? Does the tax shield from interest have the same risk as the debt that generates it (so that it should be discounted at the cost of debt) or as the firm's underlying assets (so that it should be discounted at the opportunity cost of capital)? How does leverage affect the cost of equity? Analysts wishing to value private firms must be conversant with the answers to these questions and be prepared to use an approach appropriate for the valuation case at hand.

Analysts also must be prepared to deal with the complications of nonpublic firms. In some cases, investors will require premiums for small size, the lack of a public market, and the loss of diversification that attends investment in private firms. These sorts of issues are becoming increasingly important as more investors seek to add private equity to their portfolios (Rath, 2010). However, we will argue that not every private-firm investment requires a substantial premium. As Damodaran (2009) notes, the need for a premium depends upon the investor: if an investor is well diversified, he may not demand any premiums at all.

This is the case for our subject company: we argue that our start-up does not need to offer its investor premiums for its small size or illiquidity. Instead, a public-comparables approach should be sufficient.

We do need to adjust our comparables' beta values for leverage, however. We demonstrate and justify this procedure, which is based on Harris and Pringle (1985), rather than the popular Hamada (1972) equation.

The paper proceeds as follows. Following a brief literature review, we outline the example scenario and present the basic Capital Asset Pricing Model (CAPM) approach, including the leverage adjustment. We then address some questions about our approach before considering further adjustments for size, unsystematic risk, and illiquidity. In the last section, we summarize our results and conclude.

LITERATURE REVIEW

To estimate the cost of equity for our example firm, we draw on the traditional discussions of the cost of capital, especially those from Modigliani and Miller (1958) through Harris and Pringle (1985). However, this literature applies to public firms, with observable market values for their securities. For our start-up, therefore, we must also consider factors unique to private firms, as described, for example, in Damodaran (2009).

The seminal work on the cost of capital is Modigliani and Miller ("MM," 1958). In this paper, MM present their famous Propositions I and II: that a firm's market value (and cost of capital) are independent of capital structure, and that the cost of equity rises linearly as the debt-to-equity ratio rises. These results assume no corporate income taxes, a constraint MM eliminate in dramatic fashion in 1963. However, it is also important to remember that MM assume that debt is riskless and fixed in amount, which can limit the application of extensions like Hamada's (1972; discussed below).

MM's 1958 paper only briefly considers corporate income taxes. They revise that discussion in 1963, concluding that the value of a levered firm is higher than that for an all-equity firm, since the levered firm benefits from the tax shield on its debt. Again, they assume that debt is fixed in perpetuity at amount D; that the rate of debt is independent of the degree of leverage; that firms will always be able to take advantage of their tax shields through earnings offsets, carrybacks/carry-forwards, or takeover; and that tax rates do not change. Debt adds value both because it is deductible and because it is "sure." Corporate tax shields have the same risk as the interest payments that generate them, so that discounting the perpetual stream of interest tax shields (which equals the tax rate times the interest payment, or $T_C*[\text{rate}_{debt}*D]$) at rate_{debt} gives the value increment from leverage, T_C*D. This appears to rise without limit as leverage increases.

Hamada (1972) uses MM's 1963 scenario to link corporate finance discussions of the cost of capital to the investments literature on the Capital Asset Pricing Model. Retaining MM's assumptions about fixed, perpetual, safe debt (as well as allowing for preferred dividends that are uncorrelated with the market), Hamada determines that the value of an unlevered firm's equity, scaled by its beta (an "asset" beta), equals the value of a levered firm's equity, scaled by its beta (a "levered" equity beta). Using MM's results to define an unlevered firm's equity as $(Value_{levered} - TC*D)$, a substitution results in the "Hamada equation": $\beta_{levered} = \beta_{unlevered}*[1 + (1 - tax\ rate)\dfrac{D}{E_{levered}}]$. This equation allows us to separate the operating risk of a company (reflected in $\beta_{unlevered}$) from its financing risk. It is therefore often employed to help adjust industry-comp betas for differences in leverage. However, despite its popularity, Hamada's equation can only be used properly in situations in which MM's assumptions—especially that debt is fixed and that its beta is zero—hold.

A more general approach to adjusting the cost of capital for leverage comes from the work of Miles and Ezzell (1980) and Harris and Pringle (1985). Miles and Ezzell note that the critical implication of MM's

debt assumption is not that the flows are perpetual, but that they create a constant leverage ratio over time. Thus, it is the debt rebalancing policy of the firm that drives the value of its debt tax shields. If a firm rebalances each period to keep its debt at a constant proportion of firm value, then only the first period's debt can be known with certainty (since the *future* value of the firm—and therefore of its debt—cannot be known with certainty). Thus, in contrast to MM, Miles and Ezzell discount only the first period's tax shield at the debt rate; later shields are subject to the risk of the assets, and therefore are discounted at the unlevered equity rate. This lowers the benefit of debt financing value relative to MM's approach.

Harris and Pringle (1985) come to a similar conclusion. Starting with the "textbook" weighted average cost of capital, they decompose it into two parts: the return required for operating risk, plus the tax benefits of debt financing. This decomposition implies that all tax shields—including the one for the first period—are discounted using the unlevered (asset) return, which lowers slightly their estimate of the benefits of the tax shields relative to Miles and Ezzell's. They justify this discount by pointing both to the risk that a firm may be unable to utilize its tax shield and to the risks of leverage-induced distress in general. The difference is not large, however, and the two approaches are equivalent given continuous rebalancing. (See Cooper and Nyborg, 2004, and Farber, Gillet, and Szafarz, 2007.) As Harris and Pringle point out, both approaches fall between the MM position ("considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves") and Miller's 1977 position incorporating personal taxes ("too extreme for some because it implies that debt cannot benefit the firm at all"). We will use Harris and Pringle's approach, which is advocated in Brealey, Myers, and Allen (2011).

The literature just discussed concerns public firms. Since the firm we will consider is private, however, we may need to consider adjusting the CAPM-based approach for risk factors unique to untraded firms— for example, by adding premiums for size, illiquidity, and/or lack of marketability. Damodaran (2009) discusses various approaches to estimating these premiums. (See also Rath, 2010.) Importantly, however, he notes that these sorts of premiums are not necessary for every small, illiquid firm, since the need for premiums is not a function of the issuer of a security but of its *investor*. If our firm's funding comes from well-diversified investors—as it does—then we may not need to adjust for size and liquidity at all. We will discuss this further, after presenting in the next section the basic leverage-adjusted, comparable firm approach.

EXAMPLE SCENARIO AND CAPITAL ASSET PRICING MODEL ESTIMATION

We will be estimating the cost of equity for an internet travel business owned by "Michael Powers." This start-up provides hyper-local travel-related content for communities along the northwest Pacific coast— for example, ads for hotels and restaurants, local weather, and real estate listings. (See Livingston, 2013.) All of its costs are determined as percentages of revenues; the firm has no financial or operating leverage. Initial funding for the business comes from the owner's father, "Joseph," who will receive 10% of monthly revenue, indefinitely, as compensation. Joseph knows it may take ten years or more for him to recoup his contribution, but he is well diversified and is not investing more than he can afford to lose.

We will begin our estimation of the cost of equity (k_e) for Michael's business using the Capital Asset Pricing Model (CAPM):

$$k_e = r_f + \beta * [E(R_M) - r_f], \tag{1}$$

where r_f is the return on the risk-free asset, $E(R_M)$ is the expected return on a (theoretical) value-weighted portfolio of all the risky assets in existence (the "market"), and β (beta) represents the firm's sensitivity to systematic (market) risk. The CAPM is an ex ante, one-period model that assumes that all investors

are well diversified and share all relevant expectations, and that markets are frictionless. We clearly must make many assumptions to make this model operational.

First, since no asset is truly risk free, we must find a proxy for r_f. It is customary to use a U.S. Treasury security's yield, but there have been two schools of thought on which Treasury rate to choose. (See Table 1.) One camp prefers the shortest-term T-bill rate, since these short-term assets have the least inflation risk, are the most nearly risk-free assets that we have, and therefore are our best proxy for the riskless construct. Short-term rates are also consistent with the one-period horizon of the CAPM. The other camp prefers the long-term T-bond rate, which includes a maturity premium. Advocates for this position assert that, if we are looking for a benchmark for an infinitely lived asset like stock, we want to use a long-term benchmark like the 30-year Treasury. Using a short-term benchmark to price a long-term asset is like comparing apples to oranges.

Table 1: Current Debt Yields

maturity	Treasury	Corporate
3 months	-0.02	
6 months	0.03	
2 years	0.34	1
5 years	1.43	3
10 years	2.91	4
20 years		5
30 years	4.28	

This table provides Treasury and corporate yield data (source: http://www.bondsonline.com; accessed 7/19/11). We will use this data to help estimate the inputs to the Capital Asset Pricing Model.

There are also intermediate positions. Pinto, *et al.*(2010) call the 20-year T-bond a "reasonable" choice, and note that Ibbotson's long-term bond yield is based on a portfolio of bonds with terms averaging 20 years. Recently, even the 10-year T-note has become a popular benchmark, given that the 30-year bond was not issued for several years in the early 2000s. Nonetheless, we agree that, "the analyst should try to match the duration of the risk-free measure to the duration of the asset being valued" (Pinto, *et al.,* 2010), so we will use the 30-year rate of 4.28% for r_f.

The next input to the CAPM is either the expected return on the market($E(R_M)$) or the market risk premium (the quantity ($[E(R_M) - r_f]$). Whichever approach we take, we are forced to use a proxy for the CAPM's "market" construct, which specifies a portfolio of *every risky asset in the world.* Historically, the S&P500 was practitioners' proxy of choice, although the broader Dow Jones/Wilshire 5000 is now often used.

If we choose to estimate $E(R_M)$ directly, we can use a summary of analysts' opinions about the next year's return on our proxy index. However, "analysts tend to be unduly optimistic in their earnings forecasts" (Brealey, Myers, and Allen, 2006), and projections of future market returns can be difficult for students to find. They therefore may turn to historical data, such as that tabulated by Ibbotson and Associates. (Table 2 provides some of the recent Ibbotson data.) This data is widely used and readily available. The Ibbotson data in Table 2 is broken down by asset type and is reported as geometric averages, in contrast to the theory (all risky assets) and assumptions (one-period horizon) of the CAPM. Nonetheless, at least with regard to the use of geometric averages, we note that practitioners often use geometric premiums when estimating equity returns, given that the lower geometric rates have been closer to alternative estimates based on economic theory. (See Pinto, *et al.*, 2010, for their discussion of

the "equity premium puzzle" and their assertion that use of geometric averages is "increasingly preferred" and a "mainstream" choice.)

Table 2: Selected Market Data

Metric	Benchmark/ Asset Class	Period	Value	Source
	small company stocks	1926-2009	11.9%	(1)
		1926-1999	17.7%	(2)
		1925-2004	12.7%	(2)
	large company stocks	1926-2009	9.8%	(1)
		1926-1999	13.0%	(2)
		1925-2004	10.4%	(2)
compound annual return	long-term government bonds	1926-2009	5.4%	(1)
		1926-1999	5.6%	(2)
		1925-2004	5.4%	(2)
	Treasury bills	1926-2009	3.7%	(1)
		1926-1999	3.8%	(2)
		1925-2004	3.7%	(2)
	inflation	1926-2009	3.0%	(1)
		1926-1999	3.2%	(2)
		1925-2004	3.0%	(2)
	common stocks		4.1%	(3)
nominal average annual return	government bonds		5.2%	(3)
	Treasury bills		11.7%	(3)
		1926-1987	7.7%	(12)
	common stocks		1.1%	(3)
real average annual return	government bonds		2.3%	(3)
	Treasury bills		8.5%	(3)
	Treasury bills	1900-2008	7.1%	(4)
historical average arithmetic premium for	Treasury bonds	1900-2007	6.5%	(5)
common stock	Treasury bills	1900-2007	7.4%	(5)
		1927-2011	7.94%	(6)
			5.5%	(7); (8)
expected risk premium			7.0%	(9)
			5.6%	(10)
			5% - 8%	(11)

This table summarizes return and premium data from the following sources: (1) Morningstar, cited in Titman, Martin, Keown (2010); (2) Ibbotson (2004); (3) Dimson, Marsh, and Staunton (2002), cited in Brealey, Myers, and Allen (2006); (4) Dimson, Marsh, and Staunton (2002), updated in Brealey, Myers, and Allen (2011); (5) Dimson, Marsh, and Staunton (2008), in Pinto, et al. (2010); (6) Fama-French factors, quoted in Chi and Fogdall (2012); (7) Welch (2001), cited in Brealey, Myers, and Allen (2006); (8) Fernandez, et al. (2012); (9) Welch (1998), cited in Brealey, Myers, and Allen (2006); (10) Brealey, Myers, and Allen (2006); (11) Brealey, Myers, and Allen (2011); (12) Cooper and Nyborg (2004).

Pinto *et al.*'s (2010) discussion of the use of geometric averages actually applies to the equity premium, not to E(R$_M$) directly. Many economists assume that the risk premium, unlike its constituent elements, is relatively stable, so that historical <u>premiums</u> can be good estimates of investors' future requirements. Several historical estimates of the risk premium also are given in Table 2 below. Students who wish to research their own values could start at Kenneth French's website, where he makes decades' worth of market data available (see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html).

The range of values in Table 2 underscores Brealey, Myers, and Allen's (2006) comment that there is "plenty of room for argument about what the risk premium *really* is," and their associated advice: "Do not trust anyone who claims to *know* what returns investors expect." Students therefore should not attempt to identify "the" right answer. They also should avoid the temptation simply to set a high value to be conservative. (While setting a high hurdle rate does weed out poor projects, it may also preclude some good ones. A manager's job is to maximize shareholder wealth, which means leaving no positive-NPV

projects on the table. Biasing our estimates toward increasing our hurdle rate is not consistent with that objective.) Based on the data in the table, and on Brealey, Myers, and Allen's (2006) conclusion that "a range of 5 to 8 percent is reasonable for the risk premium in the United States," we have chosen to use 8% in our analysis of Michael's internet travel company.

The final input to the CAPM is beta. We cannot estimate Michael's firm's beta directly, since it has no return history. We therefore will use publicly traded firms whose businesses are comparable to his. Michael identified five companies as his closest competitors: Google (GOOG) and Yahoo (YHOO) (online search and content providers); and Priceline (PCLN), Expedia (EXPE), and Orbitz (OWW) (online travel-booking sites). Some key statistics for these firms are given in Table 3.

From Table 3, we see that the search firms are defensive (their betas are less than one), while the travel sites are aggressive ($\beta > 1$). Michael's firm is more like the travel sites, so we will assume that it also has more systematic risk than average. OWW, with its negative return on equity and nonexistent (negative) P/E, is not a useful comp. We therefore focus only on Priceline and Expedia.

Table 3: Selected Valuation Statistics for Public Comparables

	PCLN	**EXPE**	**OWW**	**GOOG**	**YHOO**
market cap	26.68B	8.29B	309.72M	194.17B	19.01B
beta	1.15	1.92	2.32	0.92	0.86
price/sales	7.75	2.36	0.41	6.16	3.16
price/earnings	47.43	20.77	N/A	23.40	17.14
total debt	575.21M	1.65B	474.09M	5.10B	40.00M
ROE	37.33%	15.52%	-30.93%	19.16%	9.14%

This table provides key statistics for public firms identified by Michael as being comparable to his online travel business: Priceline (PCLN), Expedia (EXPE), and Orbitz (OWW) are online travel booking sites. Google (GOOG) operates online search and advertising programs. Yahoo! (YHOO) is an internet content provider. (source: Yahoo! Finance: finance.yahoo.com; accessed 7/19/11).

Both of these companies are levered, while Michael's firm is not. Thus, both PCLN's and EXPE's betas include the effects of financial leverage. To estimate their unlevered (asset) betas, which will serve as our benchmarks for Michael's equity beta, we will use the following equation:

$$\beta_{asset} = \beta_{debt} * \left(\frac{D}{D+E} \right) + \beta_{equity} * \left(\frac{E}{D+E} \right). \tag{2}$$

(See Brealey, Myers, and Allen, 2011, and Harris and Pringle, 1985.) To work back to β_{asset}, though, we must have estimates for the betas and market values of the comparable firms' debt. We will assume book value approximates market value (a common assumption; see Brealey, Myers, and Allen, 2006, Chapter 19, and CFA Program Curriculum, Level I, 2007, Vols. 4 and 5). Now, if we simply assume that the debt betas are 0, we have $\beta_{assetEXPE}$ = 1.92*[$8.29B/($1.65B + $8.29B)] = 1.60, and $\beta_{assetPCLN}$ =1.15*[$26.68B/($575.21M + $26.68B)] = 1.13.

Note that assuming that the debt beta is zero does not imply that the debt is risk-free: it simply means that the debt has no *systematic* risk. Nonetheless, as Brealey, Myers, and Allen (2006) note, most corporate bonds do have market risk, since firms are more likely to default when the economy is doing poorly. We might therefore get a better estimate of Priceline's and Expedia's asset betas if we used a more realistic estimate of β_{debt}. We do not have information on the firms' specific debt issues, but we do know the rates on A-rated corporate debt. Assuming a 10-year term, we see from Table 1 that the corporate yield is about 4%. The comparable Treasury yield is 2.91%. Using our market risk premium of 8%, this implies

a debt beta of (4% - 2.91%)/(8%) = 0.14. Substituting this value into (2) for both Expedia and Priceline, we get revised asset betas of 0.14*(0.166) + 1.92*(0.834) = 1.62 and 0.14*(0.021) + 1.15*(0.979) = 1.13, respectively.

The asset betas for Priceline and Expedia are two point estimates for the *equity* beta of Michael's firm, since his firm has no leverage. We could adjust this beta toward 1.0 using Blume's (1971) adjustment:

$$adjusted\ beta = (2/3) * (raw\ beta) + (1/3) * (1.0) \tag{3}$$

or, as Ibbotson does, toward a peer mean beta (Pinto, *et al.*, 2010). However, since Michael's firm is private and may warrant higher returns than its public peers (as discussed in the next section), we will not make any of these sorts of downward adjustments toward an average.

Given the point estimates of 1.60, 1.62, and 1.13, we will (subjectively) choose 1.2 as our estimate of beta. Travel can be considered a luxury, justifying a beta greater than 1. On the other hand, Michael's firm may provide the most value to customers living in the northwest, who would be more likely to consider a vacation closer to home when the economy is down (a "stay-cation"). Therefore, we would not expect a beta *much* higher than 1.

Given these inputs, we can now solve for the cost of equity for Michael's firm as 4.28% + 1.2*(8%) = 13.88%.

COMMENTS ON OUR REBALANCING APPROACH

In this section, we consider three questions about our approach to finding Michael's firm's required equity return: whether we should have used the Hamada equation to unlever the betas for our comp firms, whether our return estimate is "too low," and whether we should have used private instead of publicly traded comps. We begin with our unlevering process.

The approach to unlevering the betas of Priceline and Expedia is the approach outlined in Brealey, Myers, and Allen (2011). (See also Cooper and Nyborg, 2004, and Pinto *et al.*, 2010.) However, many textbooks instead use the Hamada equation, which, in its original form, looks like this:

$$\beta_{levered} * E_{levered,t-1} = \beta_{unlevered} * E_{unlevered,t-1} \tag{4}$$

where the left-hand side multiplies the beta and equity value for a levered firm (with the equity valued at the beginning of the period), and the right-hand side uses the comparable values for an unlevered firm. Since Hamada's work is based on Modigliani and Miller (1963), the value of an unlevered firm's stock equals the value of a levered firm's stock, plus the value of its debt (D), less the value of the debt's tax shield (D times the marginal corporate tax rate, T_C): $E_{unlevered} = E_{levered} + D - D*T_C$. Substituting for $E_{unlevered}$ in (4) and rearranging, we get the textbook Hamada equation:

$$\beta_{levered} = [1 + \frac{D}{E_{levered}} * (1 - Tc)] * \beta_{unlevered}. \tag{5}$$

(See, for example, CFA, 2012.) However, the assumptions underlying the Hamada equation make it unlikely to apply to Michael's case.

The problem with the Hamada equation here is that it, like the Modigliani and Miller work on which it is based, assumes that debt is not rebalanced. Instead, it assumes that debt is a fixed, perpetual dollar amount. This assumption is much less likely to be true than the alternative—that debt varies with assets (constant financing proportions). Brealey, Myers, and Allen (2011) recommend the constant-proportion approach (as do Pinto, *et al.*, 2010, who call the constant-dollar assumption "typically less plausible" than constant-proportion, and Cooper and Nyborg, 2004, who say that the constant-proportion approach is "likely" to be more applicable "in most cases"). We have therefore chosen not to use to Hamada equation when unlevering our comparables' betas. (Note that MM's assumption is not inconsistent with constant proportions; it is a special case of it. The constant debt amount implies a constant leverage ratio, given the perpetual cash flows MM assume. See Miles and Ezzell, 1980. Also note that the existence of corporate taxes does not render our approach invalid: under the constant-proportion approach, taxes are considered when finding the weighted average cost of capital; they are just omitted at the first step—equation (2)—when finding the beta of the assets.)

We now consider the end result of our unlevering process: even if our approach was correct, did we end up with a cost of equity estimate that is "too low"?

Students will undoubtedly argue that Michael's company is a start-up business, whose cash flows are subject to a great deal of uncertainty. It is obviously much more "risky" than an established company like Priceline or Expedia, and investors should therefore demand much higher expected returns from it.

The problem with this argument is that it ignores the potential for investor-level diversification. While Michael's firm may have greater total risk than PCLN or EXPE, an investor in his firm may be concerned only with the nondiversifiable risk, as measured by beta. Combining an investment in a small, private company with a well-diversified portfolio of other assets means that the unique risk of the small firm will be counterbalanced by the movements of the other assets. Investors only demand compensation for risk they actually bear. Thus, well-diversified investors like Joseph need to be compensated only for the market risk of their investment, not for the firm's total risk.

Even if we consider adding premiums for the unique risks of Michael's firm, we would still note that the magnitude of our k_e estimate—even before such adjustments—is not inconsistent with returns expected for other contemporaneous assets. For example, Wilshire Associates Incorporated's "Asset Return Assumptions" for 2012 included an 11.5% expected return for venture capital, 7.5% for a diversified private-markets portfolio, 5.8% for private real estate, and 7% for timberland (all of which would include an illiquidity premium). These are all lower than our estimate for Michael's firm, even before adding any size or illiquidity premiums. Nonetheless, we will consider these premiums further in the next section.

Before concluding this section, we should note that some analysts would disagree with our using public comparables to benchmark a small private business. The lack of direct comparability, of course, drives our consideration of adjustments below. However, we wish here to consider the broader point: should we even start our analysis with public company comparables?

Our income (discounted cash flow) approach is consistent with industry practice, since "…[some] analysts, distrustful of private transaction prices, draw on the market prices of publicly traded companies in the same business, and try to adjust for differences in fundamentals" (Damodaran, 2009). Pinto, *et al.* (2010) also suggest this approach. However, analysts also use market multiples-based approaches, which may use private-company benchmarks such as those provided by subscription databases (Rath, 2010). These databases provide statistics on private-company control transactions, whose businesses may "have more in common with the young business being valued" (Damodaran, 2009). For example, Pratt's Stats' "Private Deal Update" offers "general trend information on valuation multiples and profit margins [medians] for transactions in the Pratt's Stats database" (BVR, 2012). Nonetheless, students may not

have access to expensive databases, and, even if they do, information from databases may not be easily confirmed and may not be relevant to Michael's business (Rath, 2010). For example, Damodaran (2009) points out that private firms may differ on the relative liquidity of the assets they hold, the strength and persistence of the cash flows they generate, and the probability that they will go public in the future. Each of these factors would affect a target firm's valuation multiples—and make private comps less useful.

For Damodaran (2009), private-firm comparisons works best for small businesses that "plan to stay small and private," like doctor's offices and small retail businesses. For "young companies that aspire to… reach a larger market and either go public or be acquired by a public company," he recommends public comps—firms in the same industry that have successfully navigated the early stages of their lifecycles, and whose systematic risk reflects the business risk the target firm must bear. We have taken this approach. Nonetheless, in the next section we consider some adjustments to our public comps.

EVALUATING SIZE AND PRIVATE-FIRM PREMIUMS FOR MICHAEL'S FIRM

There are at least two possible reasons that an investor's required return for Michael's firm would differ from a public company's like Priceline's or Expedia's. First, Michael's firm is much smaller than these "comparable" firms, and so may warrant a "size premium." Second, his firm is a private company, which may justify premiums for nondiversification and/or illiquidity. We consider both of these complications in this section.

Size Premium

Michael's firm's investors may require a higher rate of return than we found from the CAPM, since his is a very small firm—and smaller firms are riskier firms. We could incorporate a size premium into our required return estimate, for example, by using any of the following three approaches:

$$k_e = r_f + b_{market} * (r_{market\,factor}) + b_{size} * (r_{size\,factor}) + b_{book-to-market} * (r_{book-to-market\,factor}) \tag{6}$$

$$k_e = r_f + equity\,risk\,premium + size\,premium + specific-company\,premium \tag{7}$$

$$k_e = r_f + b_{market} * (r_{market\,factor}) + size\,premium. \tag{8}$$

These are the Fama-French three-factor model, the "build-up" model, and the "modified CAPM," respectively. To use these models, we need an estimate of the relevant size inputs. Textbooks often provide information that would allow students to derive these estimates.

For example, Brealey, Myers, and Allen (2011), in their discussion of the Arbitrage Pricing Theory, turn to the three-factor Fama-French model. The authors note that the average annual size premium from 1926-2008 was 3.6%. (Similarly, Chi and Fogdall, 2012, report a statistically significant 3.66% premium from 1927-2011.) Both the model's market factor and the book-to-market factor have higher factor returns (7% and 5.2%, respectively). Of the ten industries whose size-factor sensitivities Brealey, Myers, and Allen provide, only three are positive; computers is one of these, with a sensitivity of 0.22. (Computers is the closest of the ten industries to Michael's firm. The other industries are autos, banks, chemicals, construction, food, oil and gas, pharmaceuticals, telecoms, and utilities. The largest size-factor

loading is 0.46, for construction.) Overall, in seven of the ten industry cases, the three-factor estimate is higher than the CAPM estimate. However, that is not true for computers: there, the CAPM estimate much higher (12.8% v. 6.5%, using a market beta of 1.43 and a book-to-market beta of -0.87).

Pinto, *et al.* (2010), also provide evidence on the Fama-French factors. They report a historical size premium (from 1926) of 2.7%, although they note that the realized premium was only about half that large from 1980-2006. They therefore use a 2% premium in their application of the model to the U.S. equity market.

Pinto, *et al.* (2010) also demonstrate the incorporation of a size premium in equations (7) and (8). Using Morningstar data from 2007, they give the size premiums for the smallest size deciles as 1.67%, (decile 6), 1.62% (7), 2.28% (8), 2.70% (9), and 6.27% (10). They further break down the 10th decile into 10a (defined as having a market cap between approximately $174M and $314M), with a premium of 4.35%, and 10b ($2.3M to $314M) at 9.68%. (However, they caution that the latter's results, especially, may be tainted by firms in financial distress.)

Given this evidence, an analyst might decide that a size premium of, say, 4% was warranted for Michael's firm, giving a modified CAPM estimate of about 18% for its required return. However, we would prefer instead to have a qualitative discussion with students about the possible effect of any size factor. One introduction to such a discussion would be the following note from Pinto, *et al.* (2010):

> If the CAPM were used to develop the equity required rate of return and similar risks were anticipated for the guideline public companies as for a smaller private company being valued, a small stock premium may not be warranted...the risk would likely be captured in the betas of the guideline public companies. (page 370)

Private Company Premiums

We have used public companies as our comparable firms. Michael identified these firms as his closest competitors. However, since Michael's is a private firm, we will follow Damodaran (e.g., 2009) and consider adjusting the cost of equity for nondiversifiable risk and for illiquidity.

Damodaran suggests that investors who are exposed to the unsystematic risk of a private-company investment should gross up the market beta to find a "total beta":

$$total\ beta_i \ = \ market\ beta_i/\rho_{iM}. \tag{9}$$

The correlation coefficient between the private firm and the market, ρ_{iM}, can be estimated using a sample of comparable publicly traded firms, just as the market beta for the private firm was estimated.

Students can easily find correlations between a market index and Priceline and Expedia. For example, using the six months prior to the decision point (1/3/11-6/30/11), data from Yahoo!Finance allow us to determine that the correlations with the S&P500 are 0.44 for Priceline and 0.20 for Expedia. The average correlation is therefore 0.32 (assuming that we wish to give the companies equal weight). Grossing up Michael's firm's beta of 1.2 thus gives us a total beta of (1.2/.32), or 3.75. Using this total beta, we find a much higher k_e estimate of $4.28\% + 3.75*(8\%) = 34.28\%$.

However, we should take this result with many grains of salt. We have used only a few months' worth of data to estimate the correlations; perhaps our estimates are too low. Wilshire Associates Incorporated's "Asset Return Assumptions" for 2012 gives a correlation of 0.75 for stocks and private markets; using this correlation, total beta is 1.6 and the revised CAPM estimate is 17.1%. On the other hand, if the

relevant correlations are as low as we estimate, then a well-diversified investor like Joseph should welcome such an addition to his portfolio.

More importantly, however, we may not need an adjustment for lack of diversification at all. Private firm valuation is dependent upon the use of the valuation (IPO, litigation, taxation, etc.; Damodaran, 2009; Rath, 2010). Thus, the necessary adjustments to the CAPM k_e depend on Michael's situation—in particular, on Joseph's claim. Joseph is a retired executive of a multibillion-dollar corporation. He has been able to help Michael buy two houses, and he is able to "kiss goodbye" the start-up costs for Michael's new venture. Joseph is undoubtedly a well-diversified investor. We therefore argue that an adjustment to the CAPM beta for unsystematic risk is unnecessary. Debate of this point should be a fruitful opportunity for classroom discussion.

We take a similar stand on the illiquidity of Michael's firm. Private firms are less liquid than publicly traded firms. Damodaran (2009) discusses ways to adjust the valuations of illiquid private firms for this unattractive feature. However, he notes that not all private firms warrant adjustment. Of particular importance for us, he asserts that the necessary adjustment is a function of the nature of the investor:

> The illiquidity discount is also likely to vary across potential buyers because the desire for liquidity varies [across] individuals. It is likely that those buyers who have deep pockets and see little or no need to cash out their equity positions will attach much lower illiquidity discounts to value, for similar firms, than buyers who have less of a safety margin.

Again, we are concerned with Joseph, whom we believe to be a well-diversified investor with no immediate need for the cash he will invest in Michael's business. (We note that a focus on Joseph is consistent with the "investment" standard of value discussed by Rath, 2010.) Thus, for the same reasons discussed above for nondiversification, we have not made an adjustment for illiquidity in our analysis. Again, instructors may choose to explore this topic qualitatively with their students.

CONCLUSIONS

Estimating the cost of equity is never easy, and it can be especially challenging for private firms. In this paper, we present the example of a start-up internet travel service, all of whose expenses (including the payments to its "venture capitalist," the founder's father) are percentages of revenue. We base our estimate of our firm's systematic risk on the betas of two publicly traded firms. We then consider adjustments for small size, illiquidity, and nondiversification. We conclude that such adjustments are not necessary for our case, since we are concerned with the firm's investment value to a well-diversified investor.

Basing an estimate on comparable firms first requires identifying companies with similar business risk. The founder of our firm identified two public comps, Expedia and Priceline, to be his closest competitors. However, since both of these firms use debt, we had to adjust for financial risk by unlevering their quoted betas to find their opportunity costs of capital, or "asset betas." Given our belief that firms are more likely to rebalance their debt over time—keeping a constant leverage ratio rather than a constant dollar debt—we used the Harris and Pringle (1985) method to unlever the betas, rather than the popular Hamada (1972) equation. In addition, we considered the effect of a nonzero debt beta, another deviation from the Hamada approach. Incorporating the resulting asset beta estimate with the current long-term Treasury rate and a historical market risk premium gave us our CAPM estimate for our firm's cost of equity.

We then considered size and private company premiums. Damodaran (2009) and Rath (2010), for example, discuss the differences in risk that can attend investments in private firms, and it is common to hear people assert that private equity investments are much "riskier" than their public counterparts.

However, not every private investment deserves returns in the often-cited 30%-45% range (see, for example, Manigart, *et. al,* undated). Our firm's "venture capitalist," Michael's father, is a well-diversified investor who is willing to wait ten years to recoup his investment, and acknowledges he may never get it back at all. For angels like him, the assumptions of the CAPM may better reflect return requirements than subjective models attempting to adjust for firm-specific risk that may be irrelevant. As investment activity in the private markets continues to grow, we should resist the temptation to reflexively add premiums to the required returns estimated using systematic risk alone; especially in private markets, premiums should be based not only on the characteristics of the investment, but also on the situation of the investor.

REFERENCES

Blume, M.E. (1971) "On the Assessment of Risk," *The Journal of Finance*, vol. XXVI(1/March), p. 1-10

Brealey, R.A., S.C. Myers, and F. Allen (2011) *Principles of Corporate Finance*, 10th ed., McGraw-Hill/Irwin, Boston

Brealey, R.A., S.C. Myers, and F. Allen (2006) *Principles of Corporate Finance*, 8th ed., McGraw-Hill/Irwin, Boston

Business Valuation Resources (BVR) (2012) "Pratt's Stats Private Deal Update," Quarter 1

CFA Institute (2012) *CFA Program Curriculum Level I: Corporate Finance*

CFA Institute (2007) *CFA Program Curriculum Level I: Corporate Finance and Portfolio Management, vol. 4*

CFA Institute (2007) *CFA Program Curriculum Level I: Equity and Fixed Income, vol. 5*

Chi, J. and J. Fogdall (2012) "Integrated Equity Solutions," *Dimensional Quarterly Institutional Review*, Fourth Quarter

Cooper, I.A. and K.G. Nyborg (2004) "Discount Rates and Tax," London Business School Working Paper No. 283, August

Damodaran, A. (2009) "Valuing Young, Start-up, and Growth Companies: Estimation Issues and Valuation Challenges," working paper, May

Farber, Andre, R. Gillet, and A. Szafarz (2007) "A General Formula for the WACC: A Reply," *International Journal of Business*, vol. 12(3), p. 406-411

Fernandez, P., J. Aguirreamalloa, and L. Corres (2012) "Market Risk Premium Used in 82 Countries in 2012: A Survey with 7,192 Answers." Retrieved June 8, 2013, from SSRN: http://ssrn.com/abstract=2084213 or http://dx.doi.org/10.2139/ssrn.2084213

Hamada, R.S. (1972) "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," *The Journal of Finance*, vol. 27(2), p. 435-452

Harris, R.S. and J.J. Pringle (1985) "Risk-Adjusted Discount Rates—Extensions from the Average-Risk Case," *The Journal of Financial Research*, vol. VIII(3/Fall), p.237-244

Ibbotson, R.G. and R.A. Sinquefield (2005) *Stocks, Bonds, Bills, and Inflation Yearbook*

Livingston, L. (2013) "Pacific Northwest Travel Online," *Review of Business and Finance Studies*, vol. 4(2), p. 107-130

Manigart, S., K. De Waele, M. Wright, K. Robbie, P. Desbrieres, H.J. Sapienza, and A. Beekman (undated) "Determinants of Required Return in Venture Capital Investments: A Five-Country Study." Retrieved June 7, 2013 from: http://leg2.u-bourgogne.fr/wp/1020701.PDF

Miles, J.A. and J.R. Ezzell (1980) "The Weighted Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis*, vol. 15(3/September), p. 719-730

Miller, M. (1977) "Debt and Taxes," *Journal of Finance*, vol. 32(2/May), p. 261-275.

Modigliani, F. and M. Miller (1963) "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review*, vol. 53(3/June), p. 433-443

Modigliani, F. and M. Miller (1958) "The Cost Of Capital, Corporate Finance and the Theory of Investment" *American Economic Review*, vol. 48(3/June), p. 261-297

Pinto, J.E., E. Henry, T.R. Robinson, and J.D. Stowe (2010) *Equity Asset Valuation*, 2nd ed., John Wiley and Sons, Hoboken, N.J.

Rath, R.D. (2010) "Private Company Valuation," in *Equity Asset Valuation*, 2nd ed., John Wiley and Sons, Hoboken, N.J.

Titman, S., J.D. Martin, and A.J. Keown (2011) "An Introduction to Risk and Return," *Financial Management: Principles and Applications,* 11th ed., Pearson

ACKNOWLEDGEMENTS

BIOGRAPHY

Lynda S. Livingston is a professor of finance and the Nat S. and Marian W. Rogers Professor at the University of Puget Sound. She is also the founder of Four Horsemen Investments, a not-for-profit corporation focusing on finance education. She can be reached at the School of Business and Leadership, University of Puget Sound, 1500 North Warner #1032, Tacoma, WA 98416, llivingston@ups.edu.