INVESTMENT UNCERTAINTY AND STOCK RETURNS
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ABSTRACT

This paper theoretically investigates the effect of uncertainty about future investment on expected stock returns. Based on a real options framework, we incorporate the learning-by-doing effect to analyze the irreversible investment problem. In our investment decision framework, the timing of expansion is endogenous and results from a value-maximizing decision. In addition, there are two important implications of our framework. First, we show that an increase in the relative valuation ratio, as measured by the book-to-market ratio, raises average stock returns. This positive relationship helps to explain the value premium. Second, we investigate how uncertainty about investment affects expected stock returns. Based on the closed-form solution in our framework, we suggest that less uncertainty about investment induces lower expected stock returns.

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INTRODUCTION

Recently, a number of theorists have noted that corporate investment is critical in examining the valuation of a firm and the cross-section stock returns (Berk, Green, and Naik, 1999, Zhang, 2005, and Cooper, 2006). Meanwhile, some research finds that expansion activity and the uncertainty about investment are related (McDonald and Siegel, 1986). How the uncertainty about investment affects the dynamics of stock returns, however, remains a controversial issue. Because of irreversibility, investment decision and the value of growth options vary with the uncertainty about investment (McDonald and Siegel, 1986). According to Berk, Green, and Naik (1999), a firm has two kinds of assets: assets in place that generate cash flows now and growth options that makes positive net present value investment in the future. Thus, the average systematic risks of a firm are conditional on cash flows from existing or new projects in the subsequence periods. We suggest that if making profitable investments change a firm’s systematic risks and expected returns, varying investment uncertainty should alter the value of the firm and its return dynamics.

The goal of this paper is to relate the uncertainty about future investment to average stock returns. First, by introducing a learning-by-doing effect, we identify that investment is triggered by the relative valuation ratio, which is defined as the ratio of value of existing assets to value of the new project. Moreover, we demonstrate that the level of relative valuation ratio contains crucial information about the value of growth options and the dynamics of stock returns. We prove that investment is triggered only when the profitability of existing assets reaches an upper threshold as suggested by Cooper (2006). This implies that if a firm has idle capacity, new investment is triggered easily. Consistent with Berk, Green, and Naik (1999), our model shows that the decision to invest can change a firm’s systematic risks if investment is irreversible. We derive that if a firm’s systematic risks are conditional on assets that it has hold, the expected stock returns are higher when the firm has higher relative valuation ratios. More specifically, undertaking profitable investment helps reduce average systematic risks of the firm’s future cash flows, as suggested by Berk, Green, and Naik (1999). To finance new investment, however, we need a higher relative valuation ratio to make existing assets as profitable as new projects. Hence, when the relative valuation ratio increases, new investment becomes less profitable and thus firms face higher systematic risks as well as higher returns. In brief, our framework proposes that average stock returns increase with relative valuation ratios, as measured by the book-to-market ratio, as does the so-called value premium by means of future expansion options and the learning-by-doing effect.
Second, we examine how the uncertainty about investment affects expected stock returns in an options framework. We find that greater uncertainty about investment induces higher average stock returns. In the classical literature of investment under uncertainty (McDonald and Siegel, 1986), greater uncertainty about investment postpones the timing of expansion and increases the value of growth options. However, some research argues that when firms face financing constraint on future investment, greater volatility in cash flows reduces the value of investment options (Boyle and Guthrie, 2003). In our framework, although investment irreversibility forces firms to delay profitable investment when uncertainty is high, uncertainty about investment also destroys the value of growth options by the learning-by-doing effect. When a firm’s systematic risks are conditional on assets that it holds, greater uncertainty about investment from existing and/or new assets reduce the value of growth options and increase the corresponding average stock returns. In short, we find a positive relationship between the uncertainty about investment and the expected stock returns.

In sum, our framework is close to Cooper (2006) in that the firm’s investment decision does rely on the profitability of its assets in place and is thus path dependent. That is, the value of existing assets can affect investment decisions and the value of growth options. Moreover, when the firm’s assets in place become more profitable, the value of growth options increase and the probability that the firm undertakes investment also increases. Most importantly, average stock returns increase with the relative valuation ratio and the uncertainty about investment.

LITERATURE REVIEW

Our study relates to two areas of research on financial economics. One relates to the issue about investment under uncertainty, and the other discusses the dynamics of stock returns by means of optimal corporate investment. More specifically, our research examines the association between uncertainty about investment and stock returns. In this section, we discuss previous literature and its implications for our investigation.

To analyze the relationship between investment and uncertainty, McDonald and Siegel (1986) apply the real options model to discuss the optimal timing of investment. In that model, the firm has perpetual rights to a new project and seeks to choose the optimal investment timing that maximizes the expected payoff. They assume both the benefits from the project and investment costs follow continuous-time stochastic process, and the investment decision is independent to the financing decision. Because the expected payoff from the new project is uncertain and the investment is irreversible, the optimal corporate policy is to invest only when the project’s NPV exceeds a positive threshold. Based on their real options framework, both the value of the growth options and the investment threshold are increasing functions of the uncertainty about investment. Consistent with the McDonald and Siegel (1986) model that benefits and costs of new investment are path dependent, Hackbarth and Morellec (2006) extend this setup to allow for a linear connection between gains and costs of new expansion. They assume that after expansion the value of the firm increases by a constant fraction at a cost proportional to the valuation of new investment. According to Hackbarth and Morellec (2006), because control transactions (takeover, expansion, and disinvestment) generally create value for the firm, they can affect firm-level betas as well as stock returns.

Shleifer and Vishny (2003) and Morellec and Zhdanov (2005) apply similar linear approach to investigate the synergy from takeovers, another kind of investment. Shleifer and Vishny (2003) suggest that if two firms merge, the market value of new equity is the sum of capital stocks from target and acquiring firms. Morellec and Zhdanov (2005) extend their linear setting to allow for asymmetric information between outside investors and inside managers. They assume a part of the synergy from takeover is not observable to outside shareholders. However, investors can update their information according to the behavior of participating firms.
Recent theoretical literature have stressed the association between firm-level investment, valuation, and expected stock returns. An innovative work of Berk, Green, and Naik (1999) relates average stock returns, systematic risks, and firm properties such as firm size and book-to-market ratio. In this model, the value of the firm is composed of the value of assets in place and growth options. They suggest that making a profitable investment will reduce the average systematic risk of the firm’s cash flows in subsequent periods, which in turn leads to lower stock returns. Based on Berk, Green, and Naik (1999), further studies incorporate the costly reversibility problem into investment decisions to examine the linkage between firm-level investment and stock returns. Zhang (2005) develops a neoclassical industry equilibrium framework with aggregate uncertainty about profitability and shows that firms’ optimal investment can generate the observed value premium, if investment is costly reversible and the price of risk is countercyclical. More specifically, he demonstrates that the asymmetric convex adjustment costs of investment gives rise to cyclical behavior of value and growth betas. In an economic downturn, capital invested is riskier than growth options because it is difficult to disinvest, while growth options are as risky as assets in place in economic booms because growth firms invest more in this situation. Hence, assets in place are riskier than growth options especially in bad times.

Cooper (2006) develops a dynamic real options model to examine the relationship between the book-to-market ratio and investment that accounts for the value premium. If capital investment is largely irreversible, the book value of assets of a distressed firm remains constant but its market value falls when facing adverse profitability shocks. That is if a firm has idle physical capacity, it is very sensitive to the aggregate productivity shock resulting in higher book-to-market ratios. Its excess installed capital capacity allows it to gain from positive aggregate shocks without undertaking new costly investment, thus providing a high return to stockholders. In contrast, a low book-to-market firm would have to undertake investment to gain from positive shocks. Hence, it is less sensitive to economics shocks and has lower systematic risks. He suggests that a firm undertakes new investment only when profitability is sufficiently high. His model also shows that irreversibility of investment, not costly reversibility, is the driving force behind the value premium. In sum, our contribution is that we help fill the gap between expected stock returns and uncertainty about investment. Our framework shows that uncertainty about investment not only governs the optimal timing of expansion but also affects expected stock returns.

THE MODELS

In this paper, we apply the rational real-option approach to analyzing investment decisions under uncertainty for all-equity firms. In this static framework, uncertainty of the economy is from a complete probability space \((\Omega, F, P)\). Using a linear setting as our valuation benchmark (Berk, Green, and Naik, 1999, and Shleifer and Vishny, 2003), we develop a two assets model to investigate investment decision problems. In contrast to previous literature that is limited to only the value of new capital stocks, we argue that both the value of new capital and the value of existing capital have apparent effects on the expansion decision. In this section, we build our basic two assets model and briefly introduce the interaction between existing assets and investment.

According to Berk, Green, and Naik (1999), we assume that assets in place and new investment create the value of the firm in this framework. Moreover, investment is irreversible, so that it cannot be used for any other purpose. Managers can postpone the expansion options until new information about the valuation of existing and new capital is revealed. Hence, the investment decision can hinge on the valuation of both assets. We further assume that the all-equity firm only has one investment opportunity, but the optimal investment scale can be distinct among firms. In addition, we assume that the irreversible investment option is infinite-lived.
Moreover, we presume that productivity of existing and new capital stocks are different but can affect each other. This is the so-called learning-by-doing effect. The simplest case of learning-by-doing is when learning occurs as a side effect of the production of new capital. Given \( G_i \) and \( H_i \), which represent the present value of future cash flows per unit of existing and new capital, respectively, after investment the valuation per unit of capital can be shown as:

\[
G_i = G_i + I_i^G(G_i, H_i) \quad \text{and} \quad H_i = H_i + I_i^G(G_i, H_i) \tag{1}
\]

In equation (1), \( G_i \) represents the valuation per unit of existing assets, and \( H_i \) stands for the valuation per unit of newly investing capital. Suppose that the valuation of each asset has two components. The first factor is the present value of the future cash flows generated by their original operation, \( G_i \) and \( H_i \); the second factor is the potential extra benefits created by new investment awaiting implementation. We assert that assets in place benefit from new investment and the synergy from new investment is conditional on the valuation of existing assets. Therefore, the implicit value of each asset is dependent. In brief, if the learning-by-doing effect is under consideration, the valuations of existing and new capital stocks are related and cannot be evaluated separately. If the capital stocks of existing and new assets are \( K_i \) and \( K_i \), respectively, the value of the firm is given by

\[
V(G, H) = (K_i + K_i)(\mu + \alpha)G + (1 - \lambda - \alpha + \alpha\beta)H \tag{2}
\]

where \( \lambda = K_i / (K_i + K_i) \) referring to the book ratio and can be applied to capture the relative importance of existing and new capital stocks. We further assume the learning-by-doing effect is distinct among new and existing capital stock. In such a setting, it is easy to identify what kind of driving force, improvement on productivity of existing capital stocks or improvement on productivity of new capital stocks, is behind the investment decision. In our model, \( \alpha \) and \( \beta \) are parameters describing the improvement on productivity from expansion for existing and new capital stocks, in which \( \alpha \) is shared by both assets but \( \beta \) is only beneficial to new capital stocks. In addition, \( \alpha \) is observable to all outside investors but \( \beta \) is only observable to inside managers. From equation (2), we assert that given an investment option the productivity of these two capital stocks will change in a predictable way if both \( \alpha \) and \( \beta \) are observable. For simplicity, we do not discuss the heterogeneous investor problem in this model and assume that all investors have the same opinion about these changes. Thus the information parameters, \( \alpha \) and \( \beta \), are constant for all investors but can vary among firms to investigate heterogeneous productivity.

The source of investment uncertainty in our framework is the future cash flows generated by these two assets. Prior to investment, we assume the present value of these cash flows evolve as follows:

\[
dG/G = \mu_G dt + \sigma_G dW_G \quad \tag{3}
\]

\[
dH/H = \mu_H dt + \sigma_H dW_H \quad \tag{4}
\]

\( \mu \) and \( \sigma \) are, respectively, the drift and volatility of the growth rate of cash flows. \( W_i \) is the standard Brownian motion on \((\Omega, F, P)\). Besides, \( W_G \) and \( W_H \) are two dependent standard Brownian motions with constant correlation \( \rho \). Furthermore, by setting \( \rho < 1 \), our model captures the feature that changes in the value of existing asset can be the result of economic shocks other than those driving new investment.

When growth options are under consideration, the synergy created by the new project can be expressed as:
\[ I(G, H) = V(G, H) - GK_1 - HK = \alpha(K_1 + K)[G + (\beta - 1)H] \]  \hspace{1cm} (5)

HK is the cost of investment and it is time-varying to verify the importance of timing to investment. Once the firm undertakes new investment, it is irreversible in that the project cannot be abandoned. However, we need two additional assumptions, \( \alpha > 0 \) and \( \beta > 1 \), to make sure \( I_G = \partial I / \partial G > 0 \) and \( I_H = \partial I / \partial H > 0 \). In other words, we need the value of the firm and the value of growth options to increase with the valuation of existing and new capital stocks. Equation (5) shows that the more improvement in productivity the larger synergy that a new project can create for the company. If the synergy created by new investment is less than zero, the firm will not undertake any investment as it needs internal funds to finance new projects. This criterion is not valid, however, especially when investment is irreversible and faces uncertainty. The following proposition shows the optimal timing of investment and the corresponding value of this growth option when investment is irreversible.

Proposition 1: Suppose that the true value of the synergy parameter is \( \beta = \beta^* \). The optimal investment strategy of a firm is to expand when the relative valuation ratio, \( R = G / H \), is at or above this level

\[ R^* = (\beta^* - 1)\eta / (1 - \eta) \]  \hspace{1cm} (6)

Moreover, the corresponding value of this growth options is

\[ O(G, H) = HAR^\eta = H \left( R^* \right)^{\eta} \alpha(K_1 + K)[R + (\beta^* - 1)]R^\eta \]  \hspace{1cm} (7)

where \( \eta \) denotes the positive root of the following familiar quadratic equation

\[ \frac{1}{2} \left( \sigma_G^2 - 2\rho \sigma_G \sigma_H + \sigma_H^2 \right) \eta(\eta - 1) + (\mu_G - \mu_H) \eta + (\mu_H - r) = 0 \]  \hspace{1cm} (8)

in which \( \eta < 1 \).

As shown in Proposition 1, a firm’s optimal investment policy is governed by a constant threshold \( R^* \). The value-maximizing expansion policy is to expand when the relative valuation ratio reaches this cutoff level. This implies that new capital is valuable only when the existing capital stocks have higher profitability or there is no idle capacity problem. Our investment decision model differs from the previous studies in which assets in place do not affect the firm’s investment decisions, such as Berk, Green, and Naik (1999). However, our work is close to Cooper (2006) that the optimal timing of expansion does depend on the profitability of the firm’s existing assets. He suggests that investment is triggered only when productivity is high enough relative to the stocks of existing capital, so that the benefits of adjusting the capital stock cover the costs of doing so. Prior to investment, the value of the growth options will depend on the timing of expansion and contain uncertainty. In the following sections, we will discuss the implications of this optimal investment strategy.

THE OPTIMAL INVESTMENT STRATEGY

This section investigates the optimal investment activity of the firm derived in Proposition 1. From equation (6), we find that the firm’s investment decision involves two sources of uncertainty: the information set about improvement in productivity, and the dynamics of future cash flows. In this section, we discuss the impact of these two characteristics on optimal investment.
First, from our closed-form solution in equation (6), we find that only the unknown productivity parameter $\beta$ is critical to the timing of expansion. Our intuition is that because $\alpha$ is observable and shared by both assets, it cannot reveal any useful information to the dynamics of relative valuation ratio $R$. Hence, only the unrevealed information has impact on the optimal timing of investment. In addition, because the relative valuation ratio is non-negative, the constant investment threshold should be positive. From equation (6), we can verify that $\partial R^*/\partial \beta > 0$. That is the firm that creates a large learning-by-doing effect through investment is not eager to chase profitable investments by setting a strict threshold. Our explanation is that if the improvement in productivity is large, the firm will hold the growth options to maximize the value of waiting to invest. Because $\beta$ is not observable to the outside investors, managers will hold the growth options until existing capital has higher valuation. In brief, waiting becomes more valuable to managers because this growth option can make existing assets more valuable.

Next, we discuss how the dynamics of cash flows affect the investment threshold. Figure 1 shows some comparative static to discuss the effects of cash flow dynamics in our framework. First, we present a number of key model parameters used in our analysis. The mean and volatility of cash flows from new projects are 5% and 21%, respectively, from Ang and Liu (2004). The volatility of cash flows from existing capital stock is 29% to match the standard deviation of the annual earnings growth of U.S. corporate earnings in the period 1929 to 2001 as reported by Longstaff and Piazzesi (2004). The drift of existing capital stock is set to 12%. This implies that the average of equity return is 8.5%, consistent with the equity premium data from Campbell, Lo, and MacKinlay (1997). The appropriate discount rate is equal to 8% to keep firms holding the options. The investment ratio $1 - \lambda$ is equal to 15% from Abel and Eberly (2001). The correlation between existing and new capital stocks is set to 0.1. The improvement on productivity of new capital stocks $\beta$ is 1.3, which is consistent with the estimated reported by Hennessy (2004). Finally, because $\alpha$ is irrelevant to the investment threshold, we set it equal to one.

Figure 1: The Effect of Cash Flows’ Volatility on the Investment Threshold

Figure 1 presents the comparative static of the investment threshold. We demonstrate that cash flow uncertainty would time investment because of irreversibility. When a firm faces a higher uncertainty
about investment, proxy by $\sigma_H$, it would prefer to hold this growth option and wait to invest. This finding is consistent with the previous research that a higher level of uncertainty will increase the critical investment trigger level (Sarkar, 2000). Greater uncertainty increases the incentive to keep the growth options in order to obtain more information about future prices and market conditions. Most importantly, we find that uncertainty about profitability from existing assets also times investment. Because of learning-by-doing, the valuation of existing assets also has impact on the synergy of expansion. When the profitability of existing capital stocks contains more uncertainty, managers will set a stricter investment threshold to expand latter.

Next, Figure 2 shows the impact of the cash flows volatility on the value of growth options. We find that the higher uncertainty about profitability from existing or new capital stocks reduces the value of growth options. This finding is opposite to the real options literature that a higher level of uncertainty increases options value (McDonald and Siegel, 1986). However, according to Boyle and Guthrie (2003), if the capital market has frictions such that a firm’s investment decision is subject to its internal funds, then greater cash flow volatility reduces the value of the expansion option because the firm has to choose a suboptimal investment timing. Consistent with Boyle and Guthrie (2003), we argue that because of learning-by-doing and the assumption of an all-equity firm, the value of growth options depends on the valuation of existing and new capital stocks. Uncertainty about profitability reduces the value of a firm’s investment opportunity and makes its market value go down. Thus, waiting is still optimal when investment is irreversible, but gains from delaying expansion decrease as profitability become more uncertain.

Figure 2: The Effect of Cash Flows’ Volatility on the Value of Growth Options

This figure shows the comparative static of the value of growth options. Two driving forces are discussed here including the volatility of cash flows from existing assets (Panel A) and the volatility of cash flows from new assets (Panel B). Input parameter values are set from previous research as described in the article. Total amount of capital stocks, $K_0 + K$, is one.

THE BEHAVIOR OF STOCK RETURNS

In this section, we derive the dynamics of the value of a firm when it has options to expand. Although there are two different sources of uncertainty in our framework, we only discuss the effect of uncertainty about profitability and assume the improvement in profitability is given. First, we derive the expected
stock returns in a closed-form expression. Based on this solution, we then do some comparative static analysis.

Consistent with Berk, Green, and Naik (1999), in our framework the value of a firm has two components, assets in place and growth options. In the previous section we derive that the optimal investment activity under uncertainty and the value of the option to invest. Thus, prior to investment the firm’s intrinsic value expresses as:

$$V(G, H) = K_1 G + O(G, H)$$

(9)

where $O(G, H)$ is defined in equation (7). If we assume that there is no private information about profitability, the implied value of the firm depends on the market valuation of these two kinds of capital. Applying Ito’s lemma, we obtain the expected rate of returns in Proposition 2.

Proposition 2: Suppose that the true value of the synergy parameter is $β = β^*$. The expected rate of stock returns can be shown as:

$$E\left(\frac{dV}{V}\right) = μ_G + \frac{O(G, H)}{V}(r - μ_G) = r + \frac{λR}{λR + AR^η}(μ_G - r)$$

(10)

The first equality of equation (10) shows that the expected stock returns are the value-weighted return of two kinds of assets, existing and new capital stocks. $μ_G$ is the expected rate of return from existing assets while $r$ is the discounted normal rate of return from holding the growth option. Given that $η < 1$ from equation (8), it is easy to derive $μ_G > r$. Given that $μ_G > r$, we find that the expected stock returns decrease with the proportion of the value of growth options to the total value of the firm.

The second equality of equation (10) shows that the expected rate of return can be related to the firm’s characteristics such as the book ratio and the relative valuation ratio. Each of them accounts for the change in the expected rate of return in a predictable way. Figure 3 shows some comparative statistics to summarize these characteristics of expected stock returns prior to investment in our framework. All parameters are identical to those in the previous section. We find the expected stock returns increase as $R$ rises. Our explanation is that when $R$ increases, the value of assets in place dominates the total value of the firm. Then returns from existing assets dominate the expected rate of return. Note that the relative valuation ratio is positively related to the firm’s book-to-market ratio. The numerator of $R$, $G$, can be viewed as the firm’s book value of assets, and the denominator of $R$, $H$, is positively related to the firm’s market value of equity. Thus, if the firm has higher $R$ or book-to-market ratio, its expected stock returns are also higher. In addition, Panel A of Figure 3 shows that the average stock returns increase with the book ratio. That is if a large proportion of the firm’s capital stocks is from existing assets, its expected stock returns are also higher. Consistent with the previous research about value premiums, we find that the firm with a higher relative valuation ratio and/or higher book ratio earns higher expected stock returns.
Figure 3: The Effect of Book Ratio and Cash Flows’ Volatility on the Expected Stock Returns

This figure shows the comparative static of the average stock returns. Three driving factors are discussed here, including the volatility of cash flows from existing assets (Panel A), the volatility of cash flows from new assets (Panel B), and the book ratio, which captures the ratio of the capital stocks of existing assets to that of new assets (Panel C). Input parameter values are set from previous papers as described in the article. Total amount of capital stocks, \( K_1 + K_2 \), is one.

Panel B and Panel C of Figure 3 show that expected stock returns increase with uncertainty about investment. A higher volatility of cash flows from existing assets (Panel B) or new capital stocks (Panel C) produces higher average returns. Our explanation is that when uncertainty from investment is high, the value of growth options declines such that profits from existing assets dominate total value of the firm. Thus, assets in place mainly govern the firm’s systematic risks. In other word, when the firm faces higher uncertainty about investment, it will postpone the expansion project so that risks from existing assets contribute a lot to its systematic risks. In brief, by introducing learning-by-doing effect and irreversible investment, we find the expected stock returns are positively related to the uncertainty about investment.
CONCLUSION

Considerable research has found corporate investment can explain the conditional dynamics in expected stock returns (Zhang, 2005, and Cooper, 2006). In addition, a number of studies state that uncertainty about investment affects the timing and the amount of investment because of irreversibility (McDonald and Siegel, 1986). Yet, despite the substantial development of these literatures, it is still unclear how the uncertainty about investment affects stock returns. This paper develops a real options model to relate the value of growth options and the value of the firm to the uncertainty about investment, in which uncertainty refers to the volatility of growth rates in cash flows and the synergy from new projects. Because investment is irreversible, the uncertainty about investment affects firms’ expansion plans by changing the investment threshold. By introducing the learning-by-doing effect, the value of growth options declines with uncertainty. Our contribution is that we find a positive relationship between uncertainty about investment and expected stock returns by means of learning-by-doing.

Although our framework links asset prices to learning effects, we need some empirical research to support our theoretical findings. Another limitation of our work is that we only discuss one possible expansion option. An obvious extension of our work would analyze the more general case that the firm has many projects, in which the learning effect could alter with the number of projects. In addition, if the firm is not all-equity, debt may affect its investment decision and average stock returns. In such cases, investment would alter the distribution of future cash flows so that a firm’s ability to meet its debt payment obligations. Further analysis of this complex problem has the potential to yield additional insights.

REFERENCE


