

TESTING FOR STOCK PRICE BUBBLES: A REVIEW OF ECONOMETRIC TOOLS

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ABSTRACT

This paper presents an overview of several econometric tools available to test for the presences of asset price bubbles. For demonstrative purpose, the tools were applied to historical stock price and dividend data starting from 1871 through 2014. The earliest tools developed were Shiller's variance bound tests and West's two step procedure. Though these tools are useful in detecting asset prices, they are subject to some serious econometric issues. To address these limitations, Cointegration methods were used to detect asset price bubbles. Unfortunately, if there are collapsing bubbles, Cointegration techniques cannot identify multiple bubbles. To overcome this Phillips, Shi and Yu (2015) developed a right tailed Augmented Dickey-Fuller test. This test not only identifies multiple bubbles but also dates the starting and ending period of a bubble. Availability of such real time monitoring tool would significantly help investors, retirees, and portfolio managers to rebalance their portfolios during such bubble periods.

JEL: G12, G14

KEYWORDS: Stock Price Bubble, Cointegration, and Right Tail ADF

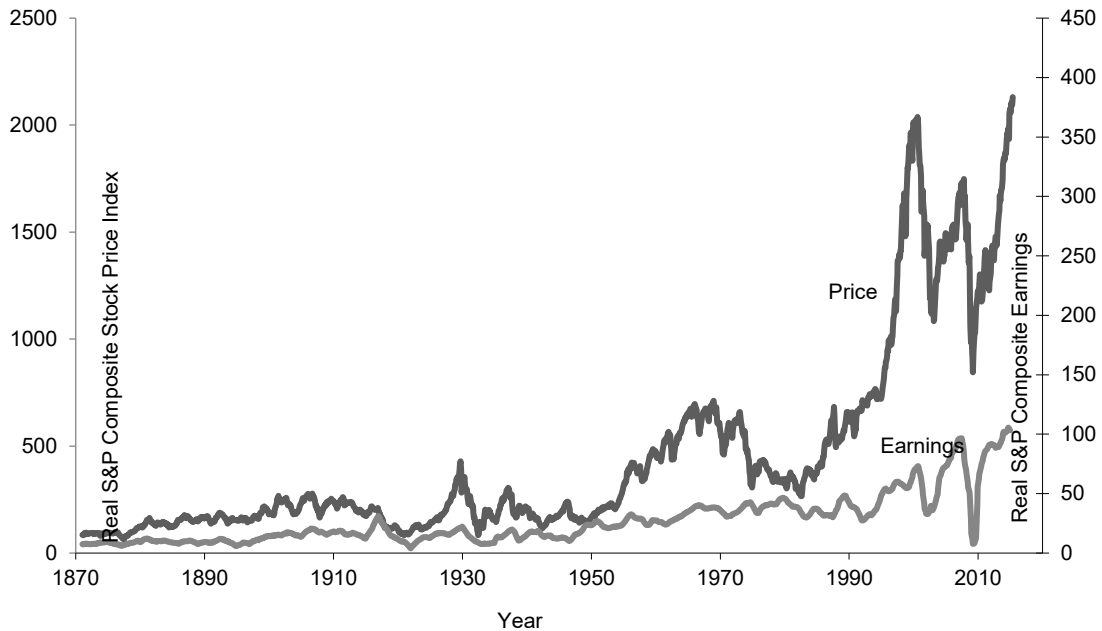
INTRODUCTION

A bubble in asset prices occurs when the present value models persistently fail to explain asset price levels greatly exceeding the value justified by fundamentals. The Finance profession is of two minds on the possibility of bubbles. On one hand the Efficient Markets School argues that asset prices are determined by the available information and this precludes the possibility of financial bubbles. They argue that when market prices deviate from the underlying fundamental values, arbitrage will force market prices to align with fundamental values. On the other hand, the Behavioral School rejects that asset prices are solely determined solely by the present value relationships. They believe investors are subject to a host of psychological biases and irrational impulses and their decisions to buy or sell may lead asset prices to deviate from their fundamental values. At the theoretical level they offer several explanations. For example, behaviorists note investors are psychologically primed to forecast current trends to continue (extrapolation bias). So that current price rises are predicted to continue to rise at the current rate. Alternatively herding behavior inclines investors to place their money in the direction of current market trends. Therefore, they conclude that in addition to fundamental factors, other psychological factors play a role in the determination of asset prices. Such theoretical disputes are best settled by examining the empirical evidence. For example, stocks prices are expected to reflect discounted future earnings.

Figure 1 shows the monthly real earnings and the real S&P 500 stock prices from 1871- 2014. While earnings were fairly stable over the whole period the S&P 500 stock prices spurted modestly in the late 1960s and early 1970s but were closely aligned with real earnings. However, beginning early 1980s stock prices rose sharply particularly relative to earnings. The sharp rise in stock prices (particularly relative earnings) in about 1997 and the subsequent decline at the end of the century led the general public and financial press to characterize the period as "the dotcom" or "the internet bubble." The upward spike in

2007 and the subsequent stock price collapse portrayed in Figure 1 led the general and financial press to characterize the period of 2007-2010 as the “Real Estate bubble”. The press attributed the recession at that time to the bursting of the bubble. The recent rise in stock prices has led the press and even academics to speculate that another bubble is forming. For example, Robert Shiller in a recent comment while expressing uncertainty claims “there is a bubble element to what we see.” (www.businessinsider.com/robert-Shiller-stock-market-bubble-2015-5).

Figure 1: S & P Price Index and S&P 500 Composite Earnings Sample: Monthly Data January 1870 - December 2014



This figure shows the monthly values of the real S&P 500 price index and the real S&P 500 composite earnings for January 1870 through December 2014. The data source is Robert Shiller’s website.

In recent years a number of econometric tools have been developed to test for financial bubbles. These tests have significant practical implications. If bubbles are present, investors, retirees, portfolio managers, regulators and policy makers, would like to detect them in order to take appropriate countermeasures. Investors, retirees, and portfolio managers would seek to rebalance their portfolios while the regulators and the policy makers could adopt appropriate policies to limit the damage to the real economy. This paper examines the econometric techniques available to detect the presence of bubbles. The original techniques focused on identifying the presence of a single financial bubble. The latter techniques enhanced our ability to spot a single bubble and in addition provided the capability to discern multiple financial bubbles and to recognize their beginning and ending points.

In the next section we review some of the previous literature and present descriptive statistics on all the variables employed in our econometric tests. We explain the methodology for the three types of bubble detection tests we utilize. The first methodology we explain is the Variance Bound Test. This test is representative of tests that investigate the consistency of elevated stock prices with the present value of the dividends model. The second test methodology examined is cointegration tests. These tests spot bubbles by studying the time series properties of stock prices and dividends. We then explain the methodology of another more sophisticated time series test which is capable of detecting multiple bubbles and to date the beginning and end of a bubble. In the following section we report the results of these three tests. In the closing section we offer our conclusions.

LITERATURE REVIEW

Shiller (1981) and LeRoy and Porter (1981) first developed variance bound to monitor the present value models under the assumption of rational bubbles. Although the purpose of these tests was to evaluate the present value of dividends model, Blanchard and Watson (1982) and Tirole (1985) among others suggested that a rejection of present value of dividend model is consistent with a bubble. Thus, the test may be used to substantiate the presence of a bubble.

Although the Variance bounds test is one of the first options developed for testing and identifying financial bubbles, we do not perform it in this paper. This is because the test has serious problems. First it tests a joint hypothesis. The test simultaneously rejects the Present Value model and thereby is unable to reject the presence of a bubble. Further, Kleidon (1986) shows the test breaks down if the data is non-stationary. Gurkaynak (2008) provides a more detailed discussion of this and other Variance bound test issues. A variation of Variance bounds test was proposed by West (1987) which can be adopted even if the data is non-stationary. Diba and Grossman (1988) note that the present value of dividend model does not allow for the possibility of a bubble starting. Thus a bubble is most likely the result of some unobserved variables. To uncover these unobserved fundamentals in the time series properties of the data Campbell and Shiller (1989) and Diba and Grossman (1988) adopted unit root and cointegration tests, respectively, to detect asset price bubbles. We will present a version of this test later in the paper.

Evans (1991) pointed out that these tests suffer from a serious limitation. He argues that these techniques cannot detect the presence of multiple bubbles in a long time series. To overcome this limitation, Phillips et al (2011) proposed a right tailed Dickey-Fuller test for detecting and dating asset price bubble. Phillips et al (2015) then generalized the right tailed Dickey-Fuller to identify and date multiple bubbles in a long time series data. We first focus on testing and dating the presence of a stock price bubble in the US. However, since US stock markets have gone through several bubbles, we also employed the generalized right tailed Dickey-Fuller tests proposed by Phillips et al (2015) to detect presence of and to date multiple bubbles. We will pay close attention to this test later in the paper.

DATA AND METHODOLOGY

The real S&P 500 total annual returns, real prices, real earnings and the real annual dividends were obtained from Robert Shiller's website for the period 1871-2014. The annual data are used to test for the presence of a bubble but monthly data from 1960 through 2014 were used to identify and date multiple bubbles. Table 1 provides descriptive statistics of annual data for the real S&P 500 prices and earnings for the periods 1871-2014 and for the end of the period 1960-2014. The mean level of Real S&P 500 Prices is substantially higher (about 95% increase) in the 1960-2014 period compared to the whole period, 1871-2014 (796.99 vs.409.26). In contrast, real Index dividends (13.17 vs. 20.06) only increased by about 52%. Obviously there is a clear miss-alignment between prices and dividends.

Table 1: Descriptive Statistics of Annual Data

| Panel A: Sample 1871 - 2014 | | | | |
|-----------------------------|--------------------------|----------------------|-----------------------|---------------------------------------|
| | Real S&P 500 Index Price | Real Index Dividends | Real S&P 500 Earnings | Ratio of Real Price to Real Dividends |
| Mean | 409.26 | 13.17 | 25.06 | 26.83 |
| Median | 232.87 | 11.63 | 18.33 | 22.09 |
| Maximum | 1975.56 | 31.45 | 92.73 | 90.89 |
| Minimum | 75.89 | 4.68 | 3.95 | 12.41 |
| Std. Dev. | 412.16 | 6.15 | 18.23 | 13.92 |
| Skewness | 2.00 | 0.72 | 1.52 | 2.23 |
| Kurtosis | 6.36 | 2.76 | 5.27 | 8.70 |
| Jarque-Bera | 160.01 | 12.36 | 84.47 | 314.55 |
| Probability | 0.00 | 0.00 | 0.00 | 0.00 |
| Panel B: Sample 1960 - 2014 | | | | |
| Mean | 796.99 | 20.06 | 43.76 | 37.81 |
| Median | 612.33 | 20.02 | 37.55 | 32.30 |
| Maximum | 1975.56 | 31.45 | 92.73 | 90.89 |
| Minimum | 290.92 | 15.31 | 24.49 | 17.70 |
| Std. Dev. | 463.23 | 3.83 | 16.90 | 16.97 |
| Skewness | 1.01 | 1.18 | 1.35 | 1.40 |
| Kurtosis | 2.72 | 4.10 | 3.96 | 4.55 |
| Jarque-Bera | 8.94 | 14.64 | 17.82 | 22.33 |
| Probability | 0.01 | 0.00 | 0.00 | 0.00 |

This Table provides descriptive statistics for each of the variables discussed in our study. It covers the entire sample period as well as the modern period. The data used are annual data.

Variance Bound Tests

As discussed previously one of the first bubble detection procedures was originated by Shiller (1981) and LeRoy and Porter (1981). We noted the reasons we do will not replicate this test. We shall employ a similar and related procedure, West’s (1987) two-step test. This test is designed to overcome the objections to the variance bound test and represents a significant advancement in bubble detection. This test avoids the problem of joint hypothesis testing by directly testing for the null hypothesis of no bubble. In addition, the test is valid even if the prices and dividends are non-stationary. To understand West’s test, it is useful to present a two period version of Shiller’s model. The present value of dividends model for the two period case is:

$$P_o = \frac{E(D_1)}{1+r} + \frac{E(P_1)}{1+r} \tag{1}$$

Where P_o is the current price, $E(D_1)$ is the expected dividend in year 1 and $E(P_1)$ is the expected price in year 1 and r is the real expected rate of return on the stock market. To apply this formula, the problem we face is that we don’t know the expected dividend stream at time 1 nor the terminal price. Shiller (1981) cleverly finessed this problem. He notes we know the past dividends and also assumes the discount rate is known and constant. Suppose we have 100 years of dividends and stock price data, then we can find a perfect forecast value at time 1 of the 100 years, p^*_1 . It is the present value of dividends years 2 through 100. (This is an approximation as it neglects dividends beyond 100. This error should be small if we truncate the sample and assign a terminal value which is average stock price of the entire sample). The perfect forecast value and the actual price differ only by the forecast error.

$$P^*_t = P_t + \varepsilon_t \tag{2}$$

If rational expectations hold then ε_t and P_t are independent. Then equation (2) implies

$$\text{Var}(P^*) = \text{Var}(P_t) + \text{Var}(\varepsilon_t) + \text{Cov}(P^*, \varepsilon_t) \quad (3)$$

and the covariance term is zero. Thus, $\text{Var}(p^*) \geq \text{Var}(P_t)$.

So if the variance of the actual stock prices exceeds the variance of the perfect forecast prices then the dividend discount model does not hold.

The dividend discount model is

$$P_t = \gamma E(P_{t+1} + d_{t+1}/I_t) \quad (4)$$

where I_t is the information available to the investor at time t and γ is the present value interest factor for the discount rate, r . That is $\gamma = 1/(1+r)$. It may be computed in a regression format with observable variables.

$$P_t = \gamma(P_{t+1} + d_{t+1}) + U_{t+1} \quad (5)$$

Where $U_{t+1} = -E(P_{t+1} + d_{t+1}) - \gamma(P_{t+1} + d_{t+1})$

West (1987) using the dividend discount model as basis, provides a two- step procedure to test for the presence of a bubble. Instead of using the Ordinary Least Squares regression, West employs an instrumental variables approach utilizing dividends as the instruments. West (1987) assumes that the dividends follow a first order autoregressive (AR (1)) process where

$$d_t = \beta d_{t-1} + V_t \quad (6)$$

He shows this implies

$$P_t = \left(\frac{\gamma\beta}{1-\gamma\beta}\right)d_t + \varepsilon_t \quad (7)$$

There are two ways to estimate $\left(\frac{\gamma\beta}{1-\gamma\beta}\right)$ in equation (7). We can use the direct result of equation (7) or the indirect result of equations (5) and (6). In the absence of a bubble the two methods will produce the same results. The null hypothesis of no bubble will not be rejected. To perform the test, we use our estimate of γ from equation (5) and the estimate of β of equation (6) and compute $\left(\frac{\gamma\beta}{1-\gamma\beta}\right)$. If this value lies in the 95% confidence interval of our estimate of the coefficient, $\left(\frac{\gamma\beta}{1-\gamma\beta}\right)$, from equation (7) we do not reject the null hypothesis of no bubble. Otherwise it is rejected and we conclude the data supports the presence of a bubble. The real S&P 500 total annual return and the real annual dividends used in our test were obtained from Robert Shiller's website for the period 1871-2014

Cointegration Tests

Both the variance bound test and West's test attempt to uncover significant deviations from a rational stock valuation model. Diba and Grossman (1988) advocate examining the time series properties of stock prices and dividends and the stability of the underlying relationship between them. Thus it is logical to ascertain whether stock prices are stationary or how much differencing is required to impose stationarity. If stock prices are more explosive over time than dividends, then this may be attributed to an asset price bubble. Thus, it is natural to test for the presence of a bubble by examining stock prices and testing for unit roots.

If we find that stock prices have unit roots while earnings do not, then it can be construed as evidence against the presence of a bubble. However, if both stock prices and earnings have unit roots then it is still necessary to probe the underlying stability of the relationship between them. One way of testing for stability in their underlying relationship is by using a statistical procedure called Cointegration. Two variables are said to be cointegrated if they share a common long run stochastic trend. However, if the relationship between earnings and stock prices becomes unstable during a period then that would support the presence of a bubble. Cointegration methodology is well suited to test for the presence of asset price bubbles. Before conducting formal tests, it is useful to review Figure 1 of real stock prices and real earnings. The plot shows that before 1982 real stock prices and real earnings appear to move together. After 1982 stock prices increased far more explosively than earnings. This raises the possibility that a cointegration test may reveal a break in the nexus between stock prices and earnings. The first step to establish cointegration is to test for the presence of unit roots. Several econometric approaches were developed in the literature to test for unit roots. For demonstrative purposes here we used the Augmented Dickey-Fuller (ADF) test to verify the presence of a unit roots in both stock prices and dividends. The second step entails testing for cointegration. To test for cointegration we used the Johansen-Juselius procedure (1988). This is accomplished by estimating the following cointegration equation:

$$P_t = a + bd_t + U_t \tag{8}$$

Where for our 2 variable case P_t is the S&P 500 Index and d_t is the dividends on the index, a and b are regression parameters and U_t is the random error term. We may rewrite the equation as

$$\Delta P_t = a + (b - 1)d_{t-1} + e_t \tag{9}$$

Where e_t is the error term of equation (9). If the variables are cointegrated then b , the slope term, will be equal to one and the null hypothesis of no cointegration is rejected. On the other hand, if b is close to zero then we will be unable to reject the null of no Cointegration. The Johansen-Juselius procedure views this equation in a matrix format. The presence of cointegration is determined by the rank of b matrix. The highest rank of b that can be obtained is n , the number of variables under consideration. If b is zero, there are no linear combinations that are stationary and so there are no cointegrating vectors. In the two variable case we consider, the matrix is a 1×1 consisting of the $b-1$ term. The rank can only be zero or one. The rank of the matrix is determined by the magnitude of the eigenvalue of the matrix. The rank of the matrix equals the number of nonzero eigenvalues. If the eigenvalue is not statistically different from zero, then the rank is zero. We do not reject the null hypothesis ($b \neq 1$); this leads to the conclusion that the variables lack Cointegration. If the eigenvalue is statistically different from zero, then we conclude the rank of the matrix equals one. This is consistent with cointegrated variables. To test this, we use the annual S&P 500 prices and the real dividend index for the period 1871-2014 from Shiller's website.

Time Dating and Multiple Bubble Tests

A method developed by Phillips et al (2011) addresses the issue of time dating financial bubbles. It constructs a right-tail Dickey-fuller test to identify the start and the end date of a bubble and provides greater power than the cointegration methodology. Still the test is unable to identify multiple bubbles. Further, the presence of multiple bubbles usually lowers the power of this testing methodology. This also applies to the West's test and Cointegration tests. This adds increased importance to the search for statistical methods capable of dealing with multiple bubbles. The Methodology of Phillips et al (2015) generalizes the analysis of Phillips et al (2011) to identify the start and end points of multiple bubbles. Both methods are based on the following reduced form equation:

$$P_t = \mu + \delta P_{t-1} + \sum \varphi_i \Delta P_{t-1} + e_t \tag{10}$$

where P_t is the price of the S&P 500, μ is the intercept, and e_t is the random error term. The δ is the key coefficient estimated by the regression. This ADF statistic (t-statistic) for this regression is not quite the same as the standard unit root ADF tests. This is because the standard unit root tests are left tail tests and while the unit root test proposed by Phillips et al (2015) are right tailed tests. We need the right tail, not the left tail values of this asymmetric distribution. Thus the critical values of the right tailed ADF statistic will differ from those used in the standard left tailed unit tests. The null hypothesis is the data contains a unit root and the alternative hypothesis postulates the presence of a mildly explosive autoregressive coefficient. Formally, the null and alternative hypotheses are:

$$H_0: \delta = 1$$

$$H_1: \delta > 1$$

Next we explain the test procedure in terms of the specifics of our dataset rather than present it in a more generalized form. With annual data it is difficult to identify multiple bubbles with periodically collapsing behavior and to precisely date when the bubble started and when it ended. Thus, to detect multiple bubbles it is important to have higher frequency data and therefore we moved from annual to monthly data. Our dataset consists of 660 monthly S & P 500 real price to dividend ratio observations covering the period of 1960 through December of 2014. We shall sample the data in fixed window sizes of 53 monthly observations, thus the first sample covers observations 1- 52. For this interval an ADF statistic is computed. Then the interval is then increased by one unit (1- 53) and an ADF_{53} is calculated. This continues until the interval covers the whole sample (1, 660). Thus 607 (660-53) ADF statistics are computed. The SADF statistic is the Supremum value of the set of ADFs calculated. A bubble occurs if the ADF exceeds the critical value of the statistic. Critical values of the SADF are set by Brownian motion of stock price movements. Critical values of the ADF statistic for each date are derived from the distribution of the ADF statistic by Monte Carlo Methods. The details may be found in Phillips et al (2011).

The Generalized SADF (GSADF) offers greater statistical power than the SADF statistic and also adds the capability of dating multiple bubbles. The first step of the GSADF test is to perform the same recursive regression as the SADF test. However, in this test instead of fixing the starting point of the recursion on the first observation, GSADF changes both the starting and ending points of the recursion. For example, a starting point of the recursion could be from 10th observation through 63rd observation. Then add one period at a time running a regression for each period until the final regression (660-53) period 607 to period 660. The GSADF for a given date is the supremum value of all the GSADFs calculated with an interval ending on that date. Since GSADF test includes more sub-samples with a flexible starting and ending windows, it does a better job of identifying multiple bubbles in the data. The distribution of the GSADF, like the SADF, are set by Brownian motion of stock price movements. This allows the use of Monte Carlo methods to find the critical values. Again Phillips et al (2015) provide details.

The methodology of dating the bubbles in real time for the GSADF test involves performing a backward SADF (BSADF) test on an expanding sample sequence. To make this concrete suppose we are considering a total sample of 200 periods. The first BSADF statistic (backwards ADF) is calculated from (200-53) 147 to 200. The second covers 146 to 200 and so on until 0 to 200. This is a special case wherein of course, the test would need to be performed on each period with changing starting and ending points. A bubble is identified when a BSADF crosses the 95% critical value from below and terminates when it crosses it from above.

RESULTS

To demonstrate the implementation of West's test we used the US stock market data. Again, the real S&P 500 total annual return and the real annual dividends were obtained from Robert Shiller's website for the period 1871-2014. The real S&P 500 price series displayed in Table 1 was discussed previously. The

dividend series is also shown in table 1. The dividends were considerably more stable than the real S&P 500 price time series as evidenced by higher standard deviation in price series in the whole period (412.16 versus 6.15). The test was conducted for the whole sample period as well as the recent period (1982-2014) where earnings and the stock prices diverged. As we discussed previously the West Test is based on a comparison of the results of a direct estimate of γ as shown in equation 7 with the results of the indirect estimate of γ and β from equations 5 and 6. The results of this are reported in Table 2. The results show the estimated coefficients of γ and β for the whole as well as for the sub sample period. Surprisingly the coefficients are stable and statistically significant for the whole and the sub-sample period. The results of estimated coefficient of equation (7) is presented in Panel C of Table 1. The coefficient is statistically significant in both periods.

Table 2: Regression Results for Equations 5 Through 7 for West Test Annual Data

| Panel A | | |
|-------------|--|---|
| | Equation 5: $P_t = \gamma(P_{t+1} + d_{t+1}) + U_{t+1}$ | |
| | WHOLE PERIOD 1871-2014 | 1982-2014 |
| | EQUATION 4 | EQUATION 4 |
| Coefficient | $\gamma = .9359$ | $\gamma = 0.9349$ |
| Std Error | 0.0145 | 0.282 |
| T-Stat | 64.33*** | 23.11*** |
| P-Value | 0.000 | 0.0000 |
| Panel B | | |
| | Equation 6: $d_t = \beta d_{t-1} + V_t$ | |
| | WHOLE PERIOD 1871-2014 | 1982-2014 |
| Coefficient | $\beta = 1.0194$ | $\beta = 1.0328$ |
| Std Error | 0.0079 | 0.0145 |
| T-Stat | 129.41*** | 71.27*** |
| P-Value | 0.000 | 0.000 |
| | Equation 7: $P_t = (\frac{\gamma\beta}{1-\gamma\beta})d_t + \varepsilon_t$ | |
| Coefficient | $(\frac{\gamma\beta}{1-\gamma\beta}) = 23.360$ | $(\frac{\gamma\beta}{1-\gamma\beta}) = 38.70$ |
| Std Error | 1.721 | 2.511 |
| T-Stat | 13.58*** | 15.41*** |
| P-Value | 0.000 | 0.000 |

*This table estimates the regressions necessary to perform the West Test. Equation 4 is a statistical representation of the present value of the dividends model. Equation 5 is an AR (1) representation of the dividends series. Finally, equation 7 combines equations 5 and 6. ***statistically significant at 1 percent*

Table 3 reports the results of testing the null hypothesis of no bubble. If there is no asset price bubble the direct estimate of the parameter in equation (7) should equal the regression estimates of equation (5) and (6) plugged into the formula $(\frac{\gamma\beta}{1-\gamma\beta})$. The γ for the whole period is 0.9359 (Equation 5) and for β is 1.019. This implies an indirect estimate of the coefficient of equation (7) is 20.7865 $(9359*1.0194/(1-0.9359*1.0194))$. The direct estimate shown in Table 2 for equation 7 is 23.36, its standard error is 1.721. This leads to a 95% confidence interval of (19.92, 26.80). Thus the confidence interval captures the value of direct estimate of equation (7) so the test fails to reject the null hypothesis of no bubbles.

Table 3: West Tests for Financial Bubbles

| Annual Data Total Sample: 1871-2014 | | | | | | |
|--|----------|---------|---|---|----------------|---------------|
| Ho: No Financial Bubble Total Sample: 1871-2014 | | | | | | |
| Period | γ | β | Indirect $(\frac{\gamma\beta}{1-\gamma\beta})$ | Direct $(\frac{\gamma\beta}{1-\gamma\beta})$ | 95% CI | Decision |
| 1871-2014 | 0.9359 | 1.019 | 20.79 | 23.36 | (19.92, 26.80) | Do Not Reject |
| 1983-2014 | 0.9349 | 1.033 | 28.04 | 38.70 | (33.68, 43.73) | Reject |

This table reports the results of the Two Step West Test for the whole period (1871-2014) and the more recent period (1983-2014). The second and third columns reports the estimates of γ and β obtained from equations 5 and 6 in Table 3 for both periods. Column 4 uses these results used to indirectly estimate the coefficient of equation 6 (for both periods). Column 5 reports the direct estimates of equation 6. If the null hypothesis of no financial bubble holds, the estimates should be equal the indirect estimates on column 5 and should be captured in the 95% confidence interval of equation 7 shown column 6.

We are most interested in the possibility of a bubble in a relatively recent period. Information from the 19th century or early or even mid-20th century probably offers little aid in detecting bubbles in the more recent period. Thus we considered the period when stock prices increased more than earnings (1982-2014). It is recent enough to shed light on the current situation and long enough to provide an adequate number of observations. The γ for the sub-sample period is 0.9339 (Equation 5) while the estimate of β is 1.0328. This implies an indirect estimate of the coefficient of equation 7, $(\frac{\gamma\beta}{1-\gamma\beta})$ of 28.04 $(0.9349*1.0328/(1-0.9349*1.0328))$. The direct estimate shown in Table 2 for equation 7 is 38.7041, with a standard error of 2.51141. This leads to a 95% confidence interval of (33.68, 43.73). Thus the confidence interval does not include the indirect estimate and the test rejects the null hypothesis of no bubbles. The West Test is consistent with the presence of a bubble or bubbles during this time period. The West test failed to detect a bubble in the whole time period. Thus our results suggest the necessity to place close attention to sub periods as the whole period may mask variation. It does not indicate when the bubble originated or when it burst. Even so, the West Test only detects bubbles but does not indicate the time of origin or the end time. Cointegration tests offer a way to pay close attention to the statistical relationship between real stock prices and real dividends during the whole period and more importantly during sub periods. If a long term equilibrium relationship between real stock prices and real dividends were to break down during a period of stock price increases, this would be consistent with the presence of a bubble. The test for cointegration ultimately tests for the presence of a common stochastic trend. If variables real stock prices and real dividends are cointegrated prior to a stock price run-up but are no longer cointegrated during the run-up, then we might infer the presence of a bubble.

Before testing for Cointegration, the stationarity of the variables must be determined. Figure 1 is clearly consistent with nonstationarity for stock prices. Still it is useful to perform more formal tests. Tests for stationarity are performed using the augmented Dickey-fuller (ADF) tests. The test allows for a drift term, a linear time trend, and for multiple period lags. The null hypothesis is that the variables are non-stationary; the alternative hypothesis is that they are stationary. The critical values for the ADF test are reported in Engle and Yoo (1987). The results of these stationarity tests are shown in Table 4. The lag length was determined by the Bayesian criterion. For both real stock prices and real dividends, the ADF test supports nonstationarity at a one per cent level of significance.

Table 4: Augmented Dickey-Fuller Unit Root Test

| Variable | Sample: Annual 1871-2014 13 Period Lag | Sample: Annual 1983-2014 1 Period Lag |
|---------------------------|---|--|
| | Augmented Dickey Fuller Test Statistic | Augmented Dickey Fuller Test Statistic |
| Real S&P 500 Stock Prices | 8.84 *** | -1.09*** |
| Real S&P 500 Dividends | 1.01*** | 1.94*** |
| Critical Value (1%) | -3.48 | -3.65 |

*This table presents the results of a formal test of the stationarity of the Real S&P500 Prices and Real S&P 500 dividends. The null hypothesis is the time series are stationary. ***Significant at 1 percent level*

Table 5 presents the results of these cointegration tests for the whole period, 1871-2014 and the 1983-2014 periods. For the whole period the Trace Statistic Test shows that for we may reject at a five per cent level of significance the null hypothesis that no eigenvalue is different from zero. Thus we may reject the null hypothesis of no cointegration at the 5% level of statistical significance. Thus real stock prices and real dividends are strongly linked. For the later portion of the sample 1983-2014 the trace tests indicate that eigenvalues are not statistically distinguishable from zero. Thus, the test fails to reject the null hypothesis that stock prices and dividends lack cointegration during this time period. This result suggests that the linkage between real S&P 500 prices and real dividends has been substantially reduced during the period of 1983-2014. The statistical evidence is consistent with the presence of a financial bubble.

Table 5: Cointegration Tests for the Presence of Financial Bubbles

| Cointegration Between | Hypothesized No. of CE(s) | Eigenvalue | Trace Statistic | 0.05 Critical Value | Probability |
|---|---------------------------|------------|-----------------|---------------------|-------------|
| Real S&P 500 Stock Prices vs. Real Dividends annual 1876-2014 | None** | 0.082 | 17.4 | 15.49 | 0.027 |
| | at most 1** | 0.038 | 5.5 | 3.84 | 0.02 |
| Real S&P 500 Stock Prices vs. Real Dividends annual 1983-2014 | None | 0.215 | 7.76 | 15.49 | 0.49 |
| | at most 1 | 0.000 | 0.005 | 3.84 | 0.94 |

*For the time period 1876-2014: Trace test indicates 2 cointegrating equations and the null hypothesis of no cointegration is not rejected. For the time period 1983 -2014: Trace test is unable to reject the null hypothesis of no cointegration. **significant at 5 percent level.*

Both West’s test and Cointegration successfully detected the presence of a bubble in the 1983-2014 period. The Cointegration test offers some improvement over the West Test in dating the bubble. The West Test only detects a bubble in the sample period and offers no guidance for the ascertaining the start of the bubble. Cointegration tests suggest an approximate starting point. The bubble starts when the cointegration relationship breaks down. Still we cannot precisely pinpoint when that occurs. Further, as we have already suggested, these methods suffer from two shortcomings. First as demonstrated by Evans (1991) these tests will detect only permanent bubbles. They fail to detect bubbles that collapse and perhaps even restart. Van Norden and Vigfusson (1998) and Hall, Psaradakis, and Sola (1999) attempted to remedy this deficiency by treating expanding and collapsing bubble as different regimes in a Markov process. Still this method cannot distinguish between a single and multiple bubbles. This is particularly important for our purposes. Both methods identified the presence of a financial bubble during 1983-2014 period. Neither method attributed the bubble to the internet, or real estate boom or both. The rise of the S&P 500 from a low of 736 to 2122 in June 2015 led the financial press to conjecture that a second equity bubble has begun. Neither of these techniques can address this issue. The SADF and GADF tests developed by Phillips et al. (2011) and Phillips et al (2014) attempts to deal with these difficulties. These tests offer the advantage of time dating the beginning and the end of a bubble. They also are capable of detecting multiple bubbles. The SADF Test establishes the start of a bubble as the first date the ADF series crosses the critical value series from below and the end of the bubble when the ADF series crosses the critical value series from above. The

estimated SADF statistic for the period and the critical values for 90%, 95%, and 99% confidences respectively are presented in Table 6.

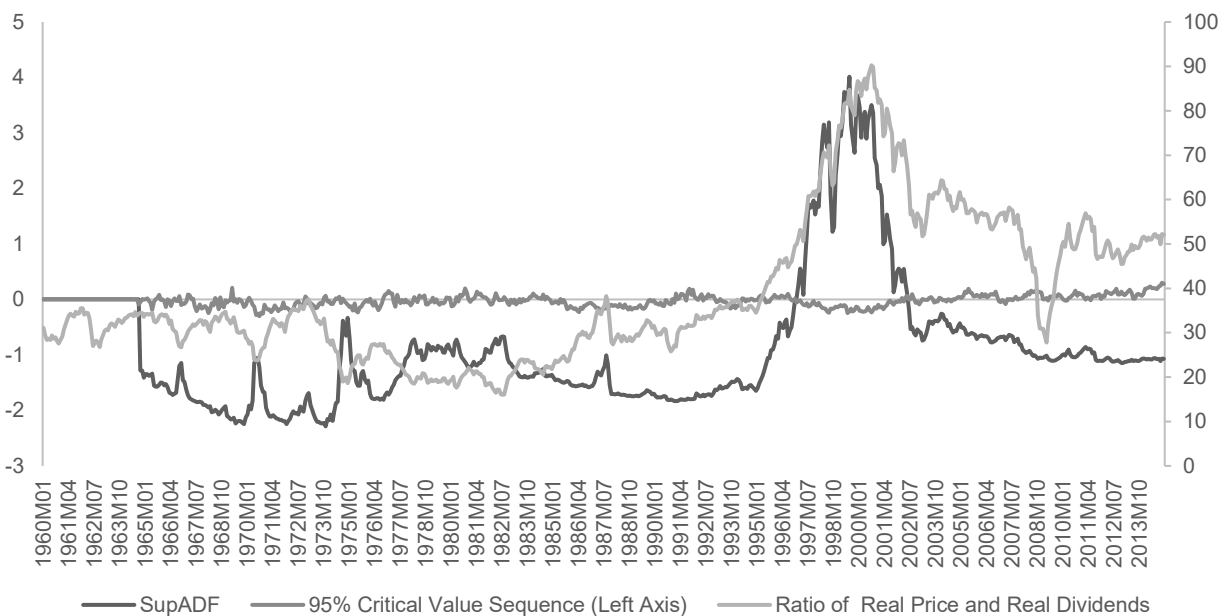
Table 6: The SADF and GSADF Test of the S&P 500 Real Price Dividend Ratio (Sample: Monthly Data 1960:01 - 2014:12)

| Test Statistic | 99% level | 95% level | 90% level |
|----------------|-----------|-----------|-----------|
| SADF | 4.01*** | 2.19 | 1.29 |
| GSADF | 4.20*** | 2.74 | 2.06 |

Critical Values for both tests for three levels of significance are derived from Monte Carlo Simulations with 1,000 replications. The Minimum window size is 53 observations. ***significant at 1 percent level

The SADF statistic of 4.01 exceeds the 1 percent right tailed critical value (2.19) indicating that the real S&P 500 price dividend ratio experienced explosive periods. To identify a specific bubble period, we compared the backward SADF statistic sequence with 95% critical value sequence obtained from Monte Carlo simulations with 1,000 replications. The results of this are plotted in Figure 2. We can see that SADF test identifies only one bubble, the dot-com bubble. The stock price bubble originated in December of 1997 and terminated in May of 2002.

Figure 2: Date Stamping Bubble Periods in the S & P 500 Price-Dividend Ratio: The SADF Test
Sample Monthly Data: January 1960 - December 2014

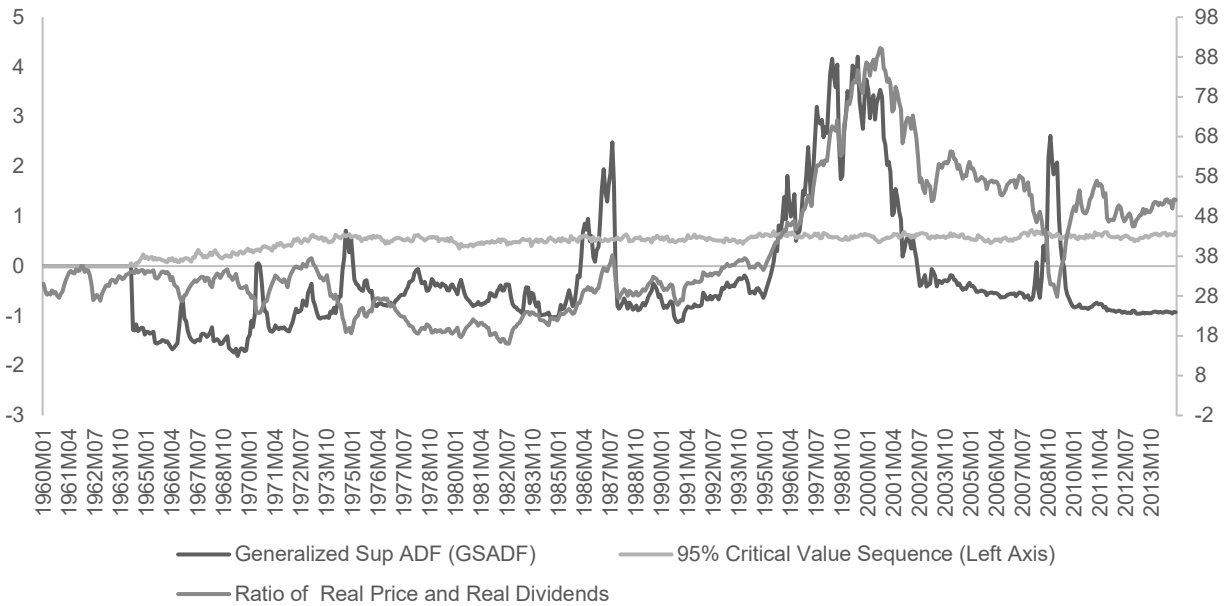


This figure records the monthly price-dividend ratio for the S&P 500, the values of the SADF statistics and the critical values for the period January 1960 through December 2014. Whenever the SADF value crosses the critical value from below this indicates the start of a bubble. Whenever it crosses from above this signifies the end of a bubble.

Table 6 also presents the results of the GSADF test. This test covers the period 1960-2014. The computational requirements of the GSADF test necessitated testing a shorter period (53 monthly observations) than used by Phillips et al. (2015). The GSADF statistic and the corresponding critical values are presented in Table 6. The GSADF statistic of 4.20 far exceeds the 1% right tailed critical value of 2.74 revealing the presence of multiple bubbles in the sample period. To identify specific bubble periods,

BSADF statistic sequence was compared to the 95% SADF critical sequence generated by Monte Carlo simulations with 1,000 replications. The plotted results are shown in Figure 3. The lower panel shows the bubble dating procedure of the GSADF test, the middle line shows the critical value sequence (left axis) and the backward SADF while the right axes measures the S&P 500 price to dividend ratio.

Figure 3: Date Stamping Bubble Periods in the S & P 500 Price-Dividend Ratio: The GSADF Test
 Sample Monthly Data: January 1960 - December 2014



This figure records the monthly price-dividend ratio for the S&P 500, the values of the GSADF statistics and the critical values for the period January 1960 through December 2014. Whenever the GSADF value crosses the critical value from below this indicates the start of a bubble. Whenever it crosses from above this signifies the end of a bubble.

The Generalized SADF identifies four bubbles between 1960 and 2014. The bubble in 1974, the 1987 black Monday bubble; the dot-com bubble and the subprime mortgage bubble. It is interesting to note that the 1974 bubble is short lived. It started in April of 1974 and ended in September of 1974. The longest identified bubble period is the dot-com bubble. The bubble started in October of 1996 and ended in November of 2001. The black Monday bubble started in November of 1986 and ended in May of 1988. Likewise, the subprime mortgage bubble started in April of 2008 and ended in May of 2009. Interestingly despite the speculation that the recent stock market surge represents a bubble, the GSADF test suggests differently. The backwards SADF statistic values since the end of the subprime mortgage crisis are well below the critical value.

CONCLUDING COMMENTS

This paper’s objective is to evaluate common econometric methods available to test for asset price bubbles. We detailed the progress of these tests which first simply tried to detect financial bubbles. To the current state wherein multiple bubbles are detectable and time datable. Therefore, availability of such real time monitoring tools would significantly help investors, retirees, and portfolio managers to rebalance their portfolios during such bubble periods. Similarly, the regulators and the policy makers could adopt appropriate policies to limit the damage to the real economy. We used historical S&P 500 index prices and dividends to employ four of the common methods used to test the presence of bubbles. The variance bound test was one of the first methods used to detect financial bubbles. The test was found to have several

problems. This led the development of the West Test. As shown in the paper it was capable of detecting financial bubbles. It could suggest the presence of bubbles but it cannot identify the beginning and ending of a bubble. Cointegration tests examine the time series properties of the data for bubbles. It suffers from the same deficiencies as the West Test that it may suggest the presence of a bubble but cannot identify the starting and ending bubble dates. Phillips et al. (2011) developed right-side unit root tests that are capable of discovering dates of asset price bubbles. However, this test does not identify multiple bubbles and hence Phillips et al. (2015) generalized their initial work by developing a procedure that would identify multiple periodically collapsing bubbles. For example, their generalized procedure identified the formation of a bubble in 1974 and before the 1987 crash. It dated the internet bubble and finally identified the real estate bubble from 2007-2010. While these are generally in line with the perception of the financial press, unlike many in the financial press find no sign of a bubble in the current stock market.

BIBLIOGRAPHY

Blanchard, O. & Watson, M. (1982). "Bubbles, Rational Expectations and Financial Markets." in Paul Wachter (ed.) *Crises in the Economic and Financial Structure*, Lexington, MA: Lexington Books, 295-315

Campbell, J. & Shiller, R. (1989). "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *The Review of Financial Studies*, 1(3), 195-228.

Diba, B. & Grossman, H. (1988). "Explosive Rational Bubbles in Stock Prices?" *American Economic Review*, 78(3), 520-530.

Engle, R & Yoo, B. (1987). "Forecasting and Testing in Co-integrated Systems." *Journal of Econometrics*, 35(1), 143-159

Evans, G. (1991). "Pitfalls in Testing for Explosive Bubbles in Asset Prices." *American Economic Review*, 81(4), 922-930.

Gurkaynak, R. (2008). "Econometric Tests of Asset Price Bubbles: Taking Stock." *Journal of Economic Surveys*, 22(1), 166-186.

Hall, S., Z. Psaradakis, Z. & Sola M., (1999). "Detecting Periodically Collapsing Bubbles: A Markov-Switching Unit Root Test." *Journal of Applied Econometrics*, 14, 143-154.

Kleidon, Allan, (1986). "Variance Bounds Tests and Stock Price Variation Models." *Journal of Political Economy*, 94(5), 953-1001.

LeRoy, S. & Porter, R., (1981). "The Present-Value Relation: Tests Based on Implied Variance Bonds." *Econometrica*, 49(3), May, 555-574.

Phillips, P.C.B., & Wu, J. (2011). "Dating the Timeline of Financial Bubbles during the Subprime Crisis." *Quantitative Economics*, 2(3), 455-491.

Phillips, P.C.B., Wu, Y. & Wu, J. (2011). "Explosive Behavior in the 1990s Nasdaq: When did Exuberance Escalate Asset Values?" *International Economic Review*, 52(1), 201-226.

Phillips, P.C.B., Shi, S. & Wu, J. (2015). "Testing for Multiple Bubbles." *International Economic Review*, 56(4), 1043-1077.

Shiller, R. (1981). "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review*, 71(3), 421-436.

Tirole, J. (1985). "Asset Bubbles and Overlapping Generations." *Econometrica*, 53(6), 1499-1528.

Van Norden, S. & Vigfusson, R (1998). "Avoiding the Pitfalls: Can Regime-Switching Tests Reliably Detect Bubbles?" *Studies in Nonlinear Dynamics & Econometrics*, 3(1), 1-22.

West, K. (1987). "A Specification Test for Speculative Bubbles." *The Quarterly Journal of Economics*, 102(3), 553-580.

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