LINKING THE SUBSTITUTION AND OUTPUT EFFECTS OF PRODUCTION TO PROFIT-MAXIMIZATION IN THE INTERMEDIATE MICROECONOMICS COURSE
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ABSTRACT

In a recent article, Thaver (2013) makes the case for including in intermediate microeconomics textbooks analysis of the substitution and output effects of a firm’s response to a change in the price of an input. In her analysis, Thaver assumes that the firm is constrained by a fixed budget for inputs, making the firm’s substitution and output effects analytically identical to the consumer’s substitution and income effects. Intermediate microeconomics textbooks typically do not assume a fixed budget for inputs when describing a firm’s profit-maximizing behavior. This paper removes the assumption of a fixed budget for inputs and provides a non-calculus presentation of substitution and output effects suitable for the intermediate course. Without this assumption, the substitution and output effects of the change in the price of an input must work in the same direction regardless of whether an input is normal or inferior, and the firm’s input demand curve, unlike a consumer’s demand curve for a good, must slope downward.

JEL: A22, D11, D24

KEYWORDS: Substitution Effect, Output Effect, Isoquants, Consumer Theory, Production Theory, Input Demand

INTRODUCTION

In a recent article in this journal, Thaver (2013) makes the case for including in intermediate microeconomics textbooks and courses analysis of a firm’s response to a change in the price of an input. In particular, she makes a case for introducing students to the substitution and output effects associated with an input price change in a way analogous to the substitution and income effects associated with a change in the price of a consumer good, emphasizing the similarities between the two. Just as a consumer’s response to a change in the price of a good can be broken into a substitution effect and an income effect, a firm’s response to an input price change can be broken into a substitution effect and an output effect. For the consumer, the substitution effect shows how he would change his purchases if his income were adjusted to leave him at his original level of well-being, and the income effect shows how he would change his purchases due to the adjustment in purchasing power at the new output prices. For the firm, the substitution effect shows how the firm would change its use of inputs if it were to continue producing the same amount of output, and the output effect shows how the firm adjusts its use of inputs when it chooses a new profit-maximizing output level.

Given the usual assumptions about production made in intermediate microeconomics courses (downward-sloping, convex-to-the-origin isoquants and profit-maximization), it can be shown that a firm’s input demand curve when there are several variable inputs must slope downward (e.g., Henderson & Quandt, 1980). This is a noteworthy result. After impressing upon students in the principles course the “law” of downward-sloping demand, intermediate courses go to great lengths to show through the analysis on income and substitution effects that the usual assumptions economists make about consumer preferences leave open the possibility of Giffen goods, goods for which a consumer’s demand curve slopes upward over some range of prices. Yet, while making what appear to intermediate microeconomics students to be
very similar assumptions about production that lead to almost identical graphical representation, the
analysis can demonstrate that input demand curves must slope downward. There can be no Giffen input.
However, while intermediate microeconomics textbooks often explain that input demand curves slope
downward and that the demand curve with several variable inputs will be more elastic than the short-run
input demand curve, they rarely relate this result to an isoquant analysis. Similarly, they do not explain
how a firm’s response to a change in the price of an input is similar to and different from a consumer’s
response to a change in the price of a good purchased. This often leaves students with the sense that what
they learned about consumer theory can be carried over to the analysis of a firm’s input demand curve.

In making her case, Thaver (2013) includes the unusual assumption that the firm is constrained by a fixed
budget for inputs. This assumption makes the firm’s response to a change in an input price identical
analytically to a consumer’s response to a change in the price of a good. Both the firm’s substitution and
output effects map directly onto the consumer’s substitution and income effects, suggesting for the firm
that substitution and output effects can work in opposite directions. However, in the rest of the treatment
of the theory of the firm, intermediate courses emphasize profit-maximization and do not assume a fixed
budget for inputs. By assuming fixed expenditure on inputs, Thaver (2013) reinforces the analysis of
consumer substitution and income effects by circling back to these concepts in the theory of the firm, but
she does so at the price of inconsistency in the coverage of firm behavior. This is a missed opportunity to
emphasize how the different assumptions about consumers and firms lead to different conclusions.

This paper offers a non-calculus presentation suitable for intermediate microeconomics textbooks and
courses of the substitution and output effects associated with a firm’s response to a change in an input
price. We show that the substitution and output effects must work in the same direction and always imply
a downward-sloping input demand curve when a firm maximizes profit without a fixed budget constraint.
For ease of exposition, this paper will focus on how a firm adjusts its use of labor when the wage rate
falls. The literature review in the next section summarizes how some intermediate microeconomics
textbooks present the topic of input demand in the long run, when there are two variable inputs. The next
section demonstrates the analysis of substitution and output effects associated with a change in the price
of a normal input, an input of which the firm would use more if it were to produce a larger quantity of
output at given input prices. We then follow the same logic to explain substitution and output effects in
the case of an inferior input, an input of which the firm would use less if it were to produce a larger
quantity of output at given input prices. Finally, there are some concluding comments on the value of this
analysis in the intermediate microeconomics course.

LITERATURE REVIEW

While most intermediate microeconomics textbooks derive a labor demand curve when both labor and
capital are variable inputs for the firm, and some emphasize that this long-run labor demand curve will be more
elastic than the short-run labor demand curve, there is little consistency in their approach to this topic.
Moreover, textbooks rarely give the same degree of analytic rigor to the presentation of a firm’s response
to a change in the wage rate as they do to a consumer’s response to a change in the price of a good.

In many textbooks (e.g. Mansfield and Yohe, 2004; Perloff, 2012; Pindyck and Rubinfeld, 2005;
Salvatore, 2003), the derivation of a firm’s long-run labor demand curve focuses on how the short-run
labor demand curve shifts when capital is also a variable input. Having demonstrated that the short-run
labor demand curve coincides with the downward-sloping portion of the firm’s marginal revenue product
of labor (MRPL) curve, these textbooks explain that a lower wage rate will not only induce the firm to use
more labor but also induce the firm to adjust its use of capital. Regardless of whether labor and capital
are complements (more of one raises the marginal product of the other) or substitutes (more of one
reduces the marginal product of the other), the adjustment upward or downward in the use of capital as
the firm uses more labor will raise the marginal product of labor, shifting the MRPL curve to the right.
This induce a further increase in the use of labor and produces a long-run labor demand curve that slopes
downward and is more elastic than the short-run labor demand curve. This approach is analytically rigorous, but fails to tie the analysis of a firm’s long-run labor demand curve back to its isoquant map in a way similar to the derivation of a consumer’s demand curve from his indifference map.

Some textbooks do derive the long-run labor demand curve or explain how the use of labor changes when the wage rate changes starting from isoquants, but coverage varies widely and is not always complete or correct. Bernheim and Whinston (2008) cover only how a reduction in the wage rate leads to a change in the least-cost input combination for producing a given amount of output, essentially limiting their analysis to the firm’s substitution effect, without going the next step to derive a labor demand curve. Similarly, Besanko and Braeutigam (2011) define normal and inferior inputs, but then show only the firm’s substitution effect and present a labor demand curve that holds output fixed. They go on to explain that the labor demand curve will shift to the right or left when output increases, depending on whether labor is a normal or inferior input. By focusing only on the substitution effect, they derive a labor demand curve that is analogous to the consumer’s compensated demand curve rather than an ordinary demand curve. Katz and Rosen (1998) explain the concepts of substitution and output effects for a firm, but incorrectly state that the output effect can work in either direction by failing to note the relation between an input being normal or inferior and whether a firm will increase or decrease output when the wage rate falls. Katz and Rosen go on to say that the net effect of the substitution and output effects must lead to a downward-sloping labor demand curve, so they provide the correct bottom-line result. Nicholson and Snyder (2007) provide a correct analysis of substitution and output effects, describing “the most common case” (p. 460) of a normal input in the body of the text. Despite delegating reference to the inferior input case to a footnote, they state in the text that they “have shown that the firm’s demand curve for an input will be unambiguously downward sloping” (p. 461). On the other hand, Nicholson (2005), a calculus-based textbook, presents substitution and output effects and then provides a calculus derivation to demonstrate that the substitution and output effects must work in the same direction regardless of whether an input is normal or inferior. However, even in this textbook the verbal summary statement about the output effect implicitly assumes a normal input.

In another calculus-based treatment, after demonstrating that an input demand curve must slope downward, Henderson and Quandt (1980) state, “There is only a substitution effect. There is no counterpart for the income effect of the consumer in the theory of the profit-maximizing producer” (p. 81). However, the consumer’s substitution effect holds utility fixed, while Henderson and Quandt do not hold output fixed. The firm’s output effect is a counterpart for the income effect of the consumer, but it does not work in the same way.

**SUBSTITUTION AND OUTPUT EFFECTS OF A CHANGE IN THE PRICE OF LABOR AS A NORMAL INPUT**

We will make the same assumptions as Thaver (2013) with the exception of not constraining total expenditure on inputs to be constant in order to be truer to the assumption of profit-maximization characteristic of the rest of the intermediate microeconomics discussion of firm behavior. We consider a perfectly competitive profit-maximizing firm that uses two variable inputs, labor (L) available at a market wage rate (P_L) and capital (K) available at a market rental price (P_K), to produce its output (Q). The analysis generalizes to more than two inputs and holds for either a price-taking firm or a firm that faces a downward-sloping marginal revenue curve, indeed for any firm that faces a marginal revenue curve that cuts its marginal cost curve from above at its profit-maximizing output level. An isoquant map for which isoquants slope downward and are convex to the origin, displaying diminishing marginal rate of technical substitution (MRTS_{L,K}) as the firm substitutes more labor for capital to produce the same amount of output, describes the technology available to the firm. We will analyze the firm’s response to a reduction in the market wage rate, holding all other parameters of the firm’s profit-maximizing decision fixed.
Suppose a firm that is initially maximizing profit finds that the wage rate falls. This change affects the firm in two ways. First, the relative prices of inputs change. Labor becomes relatively cheaper and capital relatively more expensive, so the firm would choose a different mix of inputs even if it were to produce the same amount of output. This is the firm’s substitution effect. Second, the cost of producing any amount of output falls and the firm’s marginal cost curve shifts. The new marginal cost curve leads the firm to adjust the amount of output it produces in order to maximize profit at the new input prices. This is the firm’s output effect.

Figure 1 shows a firm that initially maximizes profit by producing $Q^*$ units of output using $L_1$ units of labor and $K_1$ units of capital, the input combination for which $MRTS_{LK}$ equals the ratio of the input prices $P_L/P_K$ shown by point A where isoquant $Q^*$ is tangent to an isocost line with slope $-P_L/P_K$. When the wage rate falls to $P_L'$, the firm adjusts its profit-maximizing output level to $Q'$ units of output because its marginal cost curve has shifted in light of the new lower wage rate. The firm now employs $L_3$ units of labor and $K_3$ units of capital, the input combination along the $Q'$ isoquant for which $MRTS_{LK}$ equals the new input price ratio $P_L'/P_K$. This is shown by point B where isoquant $Q'$ is tangent to an isocost line with slope $-P_L'/P_K$. The original and new isocost lines do not necessarily intersect the vertical axis at the same point because we have not constrained the firm to spend the same total amount on inputs. The firm’s long-run labor demand curve will reflect that when the wage rate falls from $P_L$ to $P_L'$, the firm increases its use of labor from $L_1$ to $L_3$. Figure 1 implies a long-run labor demand curve that slopes downward. The question is whether this needs to be the case. To answer that question, it is useful to break the firm’s response into substitution and output effects.

The firm’s substitution effect is the change in input use that arises from the lower wage rate, if the firm were to hold its output fixed. Faced with flatter isocost lines reflecting the lower wage rate and fixed rental price of capital, the firm would use $L_2$ units of labor and $K_2$ units of capital (shown by point $A'$) for which its $MRTS_{LK}$ equals $P_L'/P_K$, the new input price ratio, if it were to continue producing $Q^*$. Because isoquants are assumed to be convex toward the origin, input combination $A'$ must contain more labor than input combination A. The substitution effect unambiguously implies that a fall in the wage rate leads the firm to increase its use of labor.

The firm’s output effect is the change in input use when the firm adjusts the amount of output it produces, holding input prices fixed at their new values. In Figure 1, the firm no longer produces $Q^*$ at the lower wage rate, but rather maximizes profit by producing $Q'$, as shown by the intersection of the firm’s new marginal cost (MC) curve with its fixed marginal revenue (MR) curve (equal to output price for a perfectly competitive firm) in the lower graph. The output effect tells us that the firm uses $L_3$ units of labor and $K_3$ units of capital at the lower wage rate (shown by point B) rather than $L_2$ units of labor and $K_2$ units of capital (shown by point $A'$).

Labor is a normal input in Figure 1, shown by the fact that the firm uses more labor at a higher output level. Moreover, we have suggested that, in this case of labor as a normal input, the firm’s MC curve shifts downward when the wage rate falls and the firm maximizes profit at a larger output level. Must this be the case? While it will seem intuitive to most intermediate microeconomics students that the answer is yes, it is worth walking through the logic in order to lay the groundwork for the case of labor as an inferior input. The total cost of producing any output level certainly goes down when the wage rate falls, but how the firm adjusts production depends on what happens to MC at the original output level. We offer two non-calculus-based explanations suitable to the intermediate course of why MC at $Q^*$ falls when the wage rate falls if labor is a normal input.
Figure 1: Substitution and Output Effects of a Decrease in the Wage Rate When Labor is a Normal Input

The firm is initially optimizing by producing $Q^*$ units of output using the input mix shown by point $A$. When the wage rate falls, the substitution effect is shown by the movement from $A$ to $A'$ in the upper graph, indicating the change in the firm's use of inputs given the new relative input prices if the firm were to continue to produce $Q^*$. The output effect is shown by the movement from $A'$ to $B$ in the upper diagram, indicating the change in the firm's input use when it adjusts its output level in light of the lower wage rate. Because labor is a normal input, the marginal cost of producing output falls when the wage rate falls, and the firm produces more output. Both the substitution and output effects work in the same direction to increase the firm's use of labor at a lower wage rate.
First, consider a simple numerical example that abstracts from the substitution effect. Suppose that producing one more unit of output requires 10 more units of capital and 5 more units of labor (both inputs normal). Marginal cost is the cost of the extra capital and extra labor needed to produce one more unit of output. When \( P_K = $10 \) and \( P_L = $6 \), the cost of producing one more unit of output is \( $130 \) \([= (10 \times $10) + (5 \times $6)]\). If the wage rate were to fall to \( P_L = $5 \), the cost of producing one more unit of output would be only \( $125 \) \([= (10 \times $10) + (5 \times $5)]\). The wage rate has gone down and so has MC.

Second, consider the geometry of the isoquant map. Marginal cost is essentially the vertical distance between the isocost lines for fixed input prices that are tangent to the isoquants corresponding to a unit increment in output. This measures MC in units of capital, the input whose price is not changing. If labor is a normal input, at any amount of labor the higher isoquant is steeper (has a higher MRTS\(_{LK}\)) than the lower isoquant. We know this because the higher isoquant has the same slope as the lower isoquant at a greater amount of labor. The isoquants and the isocost lines tangent to them are closer together at lower wage rates than at higher wage rates. Therefore, we know that, if labor is a normal input, MC falls when the wage rate falls.

Because a lower wage rate implies lower MC at the firm’s original profit-maximizing output level \( Q^* \), we know that the firm increases profit by producing a larger amount of output \( Q' \). When the firm produces more output, it uses more of all normal inputs. If labor is a normal input, the firm will unambiguously use more labor when the wage rate falls due to the output effect.

If labor is a normal input, the firm’s substitution and output effects work in the same direction and unambiguously imply a long-run labor demand curve that slopes downward.

**SUBSTITUTION AND OUTPUT EFFECTS OF A CHANGE IN THE PRICE OF LABOR AS AN INFERIOR INPUT**

How will the firm adjust its use of labor if labor is an inferior input? The analysis of the firm’s substitution effect above did not rely on whether labor is a normal or inferior input, only that the firm’s isoquants display diminishing MRTS\(_{LK}\). Therefore, even in the case of labor as an inferior input, the firm’s substitution effect implies unambiguously that the firm would use more labor to produce the same amount of output at a lower wage rate. This is shown along isoquant \( Q^* \) in Figure 2, where the firm uses \( L_1 \) units of labor and \( K_1 \) units of capital when the wage rate is \( P_L \) (point A), but would use \( L_2 \) units of labor and \( K_2 \) units of capital to produce \( Q^* \) if the wage rate were to fall to \( P_L' \) (point A').

As indicated above, to understand the firm’s output effect when labor is an inferior input, we must ask what happens to MC at the original profit-maximizing output level \( Q^* \) when the wage rate falls. First, consider once again a simple numerical example that abstracts from the substitution effect. This time, suppose that producing one more unit of output requires 10 more units of capital and 5 fewer units of labor; that is, capital is a normal input but labor is an inferior input. When \( P_K = $10 \) and \( P_L = $6 \), the cost of producing one more unit of output is \( $70 \) \([= (10 \times $10) \) - \( (5 \times $6)]\). If the wage rate were to fall to \( P_L = $5 \), the cost of producing one more unit of output would be \( $75 \) \([= (10 \times $10) \) - \( (5 \times $5)]\). The wage rate has gone down, but MC has gone up. The marginal cost of an additional unit of output is the cost of the additional capital needed less the amount saved by using less labor at the higher output level. When the wage rate goes down, the amount saved from using less labor goes down, so MC rises. While the total cost of producing any output level goes down when the wage rate falls, when labor is an inferior input the MC of producing one more unit of output goes up; the total cost curve shifts downward, but becomes steeper.

Second, consider again the geometry argument that we made above. With labor as an inferior input, at any level of labor the higher isoquant is flatter (has a lower MRTS\(_{LK}\)) than the lower isoquant for a pair of incremental isoquants. The higher isoquant has the same slope as the lower isoquant at a smaller amount
of labor. The isoquants and the isocost lines tangent to them are farther apart at lower wage rates than at higher wage rates. Therefore, if labor is an inferior input, MC rises when the wage rate falls.

Because a lower wage rate implies higher MC at the firm’s original profit-maximizing output level Q*, the firm increases profit by producing a smaller amount of output Q’. [Bear (1965) provides a more formal analysis to show that a change in the price of an inferior input will lead to a change in the profit-maximizing output level in the same direction.] When the firm produces less output, it uses less of any normal input but more of any inferior input. If labor is an inferior input, the firm will unambiguously use more labor when the wage rate falls due to the output effect.

Figure 2 summarizes the case of labor as an inferior input. The firm initially maximizes profit by producing Q* using L1 units of labor and K1 units of capital, the input combination for which MRTS_{L,K} equals the ratio of the input prices P_L/P_K, shown by point A where isoquant Q* is tangent to an isocost line with slope -P_L/P_K. When the wage rate falls to P_L’, the firm’s profit-maximizing output level falls to Q’ because its marginal cost curve has shifted up in light of the lower wage rate. The firm now employs L3 units of labor and K3 units of capital (shown by point B), the input combination along the Q’ isoquant for which MRTS_{L,K} equals the new input price ratio P_L’/P_K. The firm’s substitution effect is the change in input use from L1 units of labor and K1 units of capital (point A) to L2 units of labor and K2 units of capital (point A’) along the original isoquant Q*. The firm’s output effect is the change in input use from L2 units of labor and K2 units of capital (point A’) to L3 units of labor and K3 units of capital (point B) on the lower profit-maximizing isoquant Q’. The firm reduces its output level because its MC curve has shifted upward at Q*, so Q* no longer maximizes profit.

If labor is an inferior input, the firm’s substitution and output effects work in the same direction and unambiguously imply a long-run labor demand curve that slopes downward, just as they did in the case of labor as a normal input. Given the usual assumptions about technology and profit-maximizing behavior of firms made in intermediate microeconomics courses, there can be no Giffen input! The cases of labor as a normal and as an inferior input differ in their implications for what happens to the firm’s profit-maximizing output level, but not for what happens to labor use or for the directions of the substitution and output effects.

CONCLUSION

By relaxing Thaver’s (2013) assumption that a firm has a fixed budget for hiring inputs, we have tied the analysis of input use more closely to the assumptions about firm behavior made in intermediate microeconomics textbooks. We have also demonstrated in a way appropriate to the intermediate course that the substitution and output effects associated with the change in an input price must work in the same direction regardless of whether an input is normal or inferior. Whereas the analysis of substitution and income effects for a consumer demonstrates that these effects work in opposite directions for an inferior good and introduces the possibility of a Giffen good, the substitution and output effects for a firm must work in the same direction and always imply a long-run input demand curve that slopes downward. There can be no Giffen input!
Figure 2: Substitution and Output Effects of a Decrease in the Wage Rate When Labor is an Inferior Input

The firm is initially optimizing by producing $Q^*$ units of output using the input mix shown by point A. When the wage rate falls, the substitution effect is shown by the movement from A to $A'$ in the upper graph, indicating the change in the firm’s use of inputs given the new relative input prices if the firm were to continue to produce $Q^*$. The output effect is shown by the movement from $A'$ to B in the upper diagram, indicating the change in the firm’s input use when it adjusts its output level in light of the lower wage rate. Because labor is an inferior input, the marginal cost of producing output rises when the wage rate falls, and the firm produces less output. Both the substitution and output effects work in the same direction to increase the firm’s use of labor at a lower wage rate.
The analysis presented here may explain Thaver’s (2013) observation that no research exists on the substitution and output effects of a change in the wage rate when labor is an inferior input. The effects work in the same direction, and the differences between inputs being normal or inferior matter only if one is concerned with the effects on a firm’s output level. Indeed, one might similarly argue that the analysis of substitution and output effects of an input price change are unnecessary in the intermediate microeconomics course, as they provide no new insights to the law of downward-sloping demand. However, that would be a mistake if the goals of the intermediate course include demonstrating the coherence of microeconomic analysis and fostering the kind of analytic thinking associated with seeing how the assumptions made at the outset lead to the conclusions derived from them. In the particular case presented here, students of intermediate microeconomics should come to understand both the similarities and differences between the firm’s substitution and output effects, on the one hand, and the consumer’s substitution and income effects, on the other. They should also understand why input demand obeys “the law of downward-sloping demand,” while our usual assumptions about consumer preferences and behavior do not lead to this result.

The consumer’s and the firm’s substitution effects are analytically identical. The consumer’s substitution effect arises from the part of the consumer’s optimization principle that says that the marginal rate of substitution must equal the price ratio. The firm’s substitution effect comes from the requirement that the marginal rate of technical substitution be equal to the input price ratio. However, the second part of the consumer’s optimization principle is different from the second part of the firm’s optimization principle, so the income and output effects work differently. The income effect comes from the requirement that the consumer spend his entire income; a price decrease unambiguously leads him to a higher indifference curve and less consumption of inferior goods. The output effect comes from the part of the firm’s optimization principle that says that the profit-maximizing firm will choose the output level for which marginal cost equals marginal revenue. Depending on whether an input is normal or inferior, a decrease in the price of that input either decreases marginal cost inducing the firm to increase output or increases marginal cost inducing the firm to decrease output. The firm does not have a fixed budget to spend on inputs in the way that a consumer has a fixed budget to spend on goods. Without this careful analysis, the similarities between the indifference curve/budget line diagram and the isoquant/isocost line diagram can mislead students.

REFERENCES


**BIOGRAPHY**

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